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MOMENT AMPLIFICATION FACTOR OF BEAM-COLUMNS WITH INITIAL DEFLECTION

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In the design of slender steel beam-columns, the moment amplification factor is used to estimate the maximum bending moment. The formulas for evaluating the factor have been presented on the basis of the elastic or elastic-plastic analysis, however the initial deflection of beam-columns is not considered. This paper discusses the effect of initial deflection on the value of the moment amplification factor by performing the analytical work. The analytical model is a simply supported beam-column subjected to constant axial compressive force and end moments. First of all, the equilibrium differential equation which governs the problem is solved and the formula for calculating the bending moment is obtained. In the parametric study, magnitude of the initial deflection, the axial load ratio, the slenderness ratio and the end moment ratio are selected as the parameters. The effects of magnitude of the initial deflection and the end moment ratio on the moment amplification factor are discussed.

Keywords: Steel beam-columns, End moment ratio, Axial load ratio, Slenderness ratio, Strength formula, Analytical solution.

1 INTRODUCTION

In the design formulas of the steel beam-columns described in AIJ Recommendation for Limit State Design of Steel Structures (hereinafter referred to as LSD guideline) (AIJ 2010), when the maximum bending moment occurs between supporting points, the bending moment is evaluated by using the moment amplification factor a_m. In LSD guideline, the moment amplification factor ${}_d a_m$ is adopted as the factor at the maximum strength in the inelastic state (Sakamoto *et al.* 1968). On the other hand, the moment amplification factor ${}_e a_m$ can be analytically calculated for linear elastic steel beam-columns. These factors a_m, ${}_d a_m$ and ${}_e a_m$ have been obtained by analysis without consideration of the initial deflection. The effect of the initial deflection on the strength and behavior of compression members have been discussed (AIJ 1992), there are no studies discussing the effects of initial deflection of the beam-columns on the moment amplification factor. Objective of this study is to clarify the dimensionless factors, such as the amount of the initial deflection, end moment ratio, axial force ratio and slenderness ratio, which govern the amplification factor and the effects of these factors on the amplification factor.

2 ANALYSIS

2.1 Problem Setting

As shown in Figure 1, the analytical model is the hinged-end beam-column subjected to constant axial compressive force N and end moments M_1 and M_2(=κM_1, $|\kappa|\leqq 1$). The coefficient κ is the end moment ratio. The end moment M_1 is numerically larger than M_2. The value of κ is positive when the member is bent in reverse curvature. Boundary conditions are pinned supports at both ends. In this paper, the moment amplification factor a_m (=M_{max}/M_1:M_{max} is the maximum bending moment along the whole length of the member) is analytically calculated when there is an initial deflection.

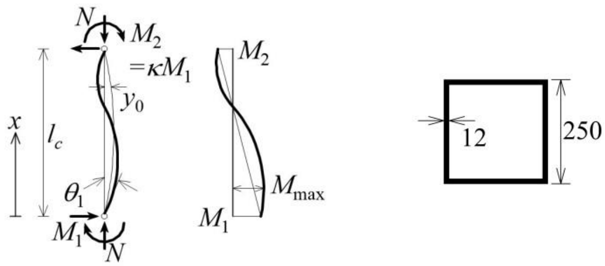


Figure 1. Analytical model.

2.2 Equilibrium Differential Equations and the Solutions

The governing equilibrium differential equation is the following equation, Eq. (1) (see Figure 1).

$$EIy_1'' + Ny_1 = -\left(1-\frac{1+\kappa}{l_c}x\right)M_1 - Ny_0 \tag{1}$$

where E is the Young's modulus, I is the moment of inertia, N is the compressive force, y_1 is the deflection, and y_0 is the initial deflection.

The initial deflection y_0 is assumed to be a sine series, Eq. (2).

$$y_0 = \sum_{j=1}^{n} c_j \sin\left(\frac{j\pi x}{l_c}\right) \tag{2}$$

Letting x/l_c be ξ, the solution of Eq. (1) can be obtained as follows:

$$y_\xi = \frac{M_1}{N}\left[\begin{array}{l} \cos\alpha\xi + \left(-\dfrac{\kappa+\cos\alpha}{\sin\alpha}\right)\sin\alpha\xi + \left\{(1+\kappa)\xi - 1\right\} \\ -\dfrac{N}{M_1}\displaystyle\sum_{j=1}^{n} c_j \dfrac{\alpha^2}{\alpha^2 - j^2\pi^2}\sin(j\pi\xi) \end{array}\right] \tag{3}$$

In the above equation, Eq. (3), the axial load ratio n_y and the normalized slenderness ratio λ_c are defined by the following equation, Eq. (4).

$$n_y = \frac{N}{N_y} \quad (N_y = A\sigma_y) \qquad \lambda_c = \sqrt{\frac{N_y}{N_e}} \quad \left(N_e = \frac{\pi^2 EI}{l_c^2}\right) \tag{4}$$

where N_y is the yield axial force, A is the cross-sectional area, and σ_y is the yield stress.

The bending moment at ξ, $M(\xi)$ is expressed by the next equation, Eq. (5),where $M = -EIy_1''$.

$$\frac{M(\xi)}{M_1} = \cos\left(\pi\sqrt{n_y\lambda_c^2}\,\xi\right) - \frac{\kappa + \cos\left(\pi\sqrt{n_y\lambda_c^2}\right)}{\sin\left(\pi\sqrt{n_y\lambda_c^2}\right)}\sin\left(\pi\sqrt{n_y\lambda_c^2}\,\xi\right) + \sum_{j=1}^{n}\frac{Nc_j}{M_1}\frac{j^2}{j^2 - n_y\lambda_c^2}\sin(j\pi\xi) \tag{5}$$

The value of $M_{\max}/M_1$ calculated by using the above equation becomes the moment amplification factor a_m. According to the above equation, a_m is related to the end moment ratio κ, $n_y\lambda_c^2$, and Nc_j/M_1. Nc_j/M_1 is expressed by n_y, c_j and η by modifying the equation of M_1. The dimensionless amount η is defined by Eq. (6), where M_y is the yield moment.

$$\eta = \frac{M_1}{M_y} \tag{6}$$

2.3 Moment Amplification Factor (Elastic and Inelastic)

Eq. (7) shows the moment amplification factor ${}_ea_m$ obtained by the elastic analysis in case that there is no initial deflection, and Eq. (8) shows the moment amplification factor ${}_da_m$ at the maximum strength in a inelastic state prescribed in LSD guideline.

$$\begin{aligned} {}_ea_m &= 1 & &\left(\kappa \ge -\cos(\pi\sqrt{N/N_e})\right) \\ &= \frac{\sqrt{1+\kappa^2+2\kappa\cos\left(\pi\sqrt{N/N_e}\right)}}{\sin\left(\pi\sqrt{N/N_e}\right)} & &\left(\kappa < -\cos(\pi\sqrt{N/N_e})\right) \end{aligned} \tag{7}$$

$$\begin{aligned} {}_da_m &= 1 & &\left(n_y\lambda_c^2 \le 0.25(1+\kappa)\right) \\ &= \frac{1-0.5(1+\kappa)\sqrt{N/N_e}}{1-N/N_e} & &\left(n_y\lambda_c^2 > 0.25(1+\kappa)\right) \end{aligned} \tag{8}$$

2.4 Analytical Parameters

We selected the initial deflection c_1 (only sine half wave is assumed), the end moment ratio κ, the axial load ratio n_y, and η in Eq. (6) as the analytical parameters, and they vary as follows:

1) $c_1 = \pm l_c/1500$, $\pm l_c/1000$, $\pm l_c/500$

2) κ =-1.0, -0.5, 0, 0.5, 1.0

3) n_y=0.1, 0.3, 0.5, 0.7

4) η =1, 2/3

The cross section used in this analysis is a square steel tube, the width of the cross section is 250 mm, and the plate thickness is 12 mm (see Figure 1). The yield stress σ_y is 325 N/mm^2.

3 RESULTS AND DISCUSSION

3.1 Moment Amplification Factor

3.1.1 *Moment amplification factor a_m, ${}_ea_m$, ${}_da_m$*

Figure 2 shows the relationship between $n_y\lambda_c^2$ and moment amplification factors a_m, ${}_ea_m$, ${}_da_m$. The moment amplification factor a_m is calculated in case of $c_1 = l_c/1000$, $n_y = 0.5$ and $\eta = 1$. In this figure, marks ●, +, □, × and ○ indicate the end moment ratio κ is -1, -0.5, 0, 0.5 and 1, respectively. The values of ${}_ea_m$ and ${}_da_m$ are shown by solid lines and ${}_da_m$ is shown by the dotted line, respectively.

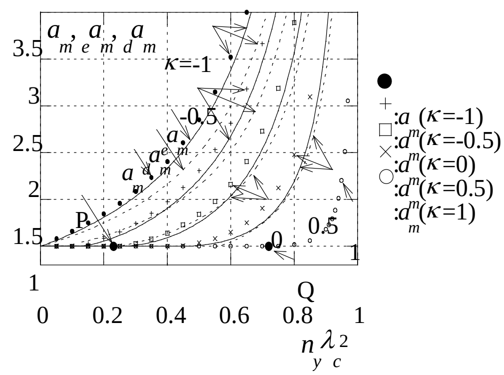


Figure 2. Moment amplification factor ($c_1 = l_c/1000$, $n_y = 0.5$, $\eta = 1$).

According to this figure, a_m is larger than ${}_ea_m$, however the relations with $n_y\lambda_c^2$ are almost same. When κ is equal to 1, ${}_ea_m$ is always 1 and the maximum bending moment occurs at the end of the beam-columns. However without initial deflection and in the range of $n_y\lambda_c^2 \geqq 0.72$, the moment amplification factor becomes larger than 1 (point Q in Figure 2). When the member is subjected to moment at only one end ($\kappa = 0$) and there is no initial deflection, the value of ${}_ea_m$ becomes 1 in the range of $n_y\lambda_c^2 \leq 0.25$. However when there is an initial deflection, ${}_ea_m$ is greater than unity in the range of $n_y\lambda_c^2 > 0.23$ (point P, in Figure 2).

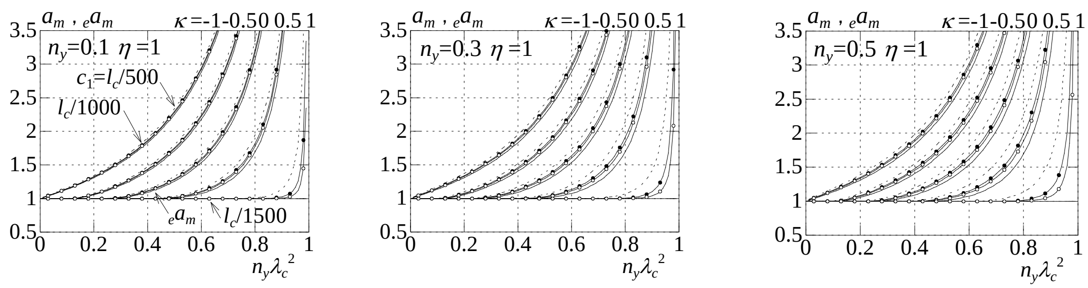
3.1.2 *Influence of initial deflection c_1*

Figure 3 shows the influence of the initial deflection c_1 on the moment amplification factor a_m. Figure 3 (a), (b), and (c) show the cases where the axial force ratios n_y are 0.1, 0.3, and 0.5, respectively. In the figure, $c_1 = l_c/1500$, $l_c/1000$ and $l_c/500$ are indicated by ○, ●, and dotted lines, respectively, and ${}_ea_m$ is indicated by a solid line.

Regardless of the value of the end moment ratio κ, the moment amplification factor a_m become large as the initial deflection c_1 increases.

3.2 $a_m/{}_ea_m - n_y\lambda_c^2$ Relations

To compare the moment amplification factors a_m when the initial deflection is present and absent, we set the value of $a_m/{}_ea_m$ as an indicator and examine the influence of two parameters that affect the value $a_m/{}_ea_m$.

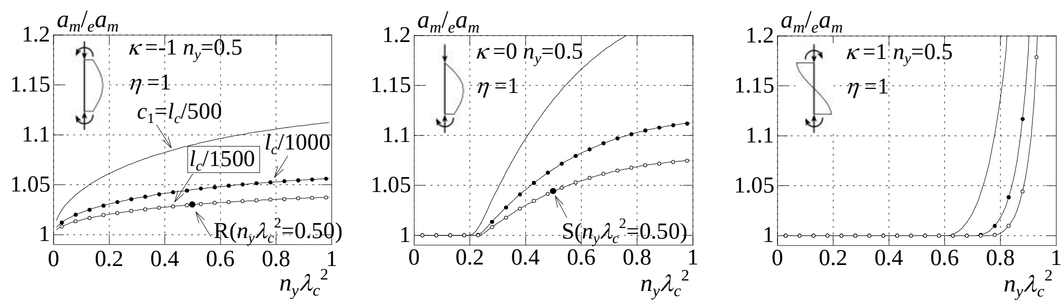


(a) $n_y = 0.1$ (b) $n_y = 0.3$ (c) $n_y = 0.5$

Figure 3. a_m-$n_y\lambda_c^2$ relations (influence of initial deflection c_1).

3.2.1 *Influence of initial deflection c_1*

Figure 4 shows the influence of the initial deflection c_1 on the value of $a_m/_ea_m$. Figures 4(a), 4(b) and 4(c) show the cases where the end bending moment ratio κ is -1, 0, and 1, respectively. In the figure, the cases of c_1 = l_c/1500, l_c/1000 and l_c/500 are indicated by ○, ● and solid line, respectively.



(a) κ=−1 (b) κ = 0 (c) κ =1

Figure 4. $a_m/_ea_m$-$n_y\lambda_c^2$ relations (influence of initial deflection c_1).

According to Figure 4(a), which is in case of uniform bending, the values of $a_m/_ea_m$ are always larger than 1.0, and the value of $a_m/_ea_m$ also increases as the initial deflection c_1 increases. In case of c_1 = l_c/1500, which is a value of the management tolerance for steel columns (AIJ 2014), the value of $a_m/_ea_m$ is at most 1.04 when the axial load ratio n_y is 0.5, and the initial deflection c_1 has little influence on the value of $a_m/_ea_m$. When $n_y\lambda_c^2$ is about 0.5, the value of $a_m/_ea_m$ is about 1.03 (point R, in Figure 4(a)).

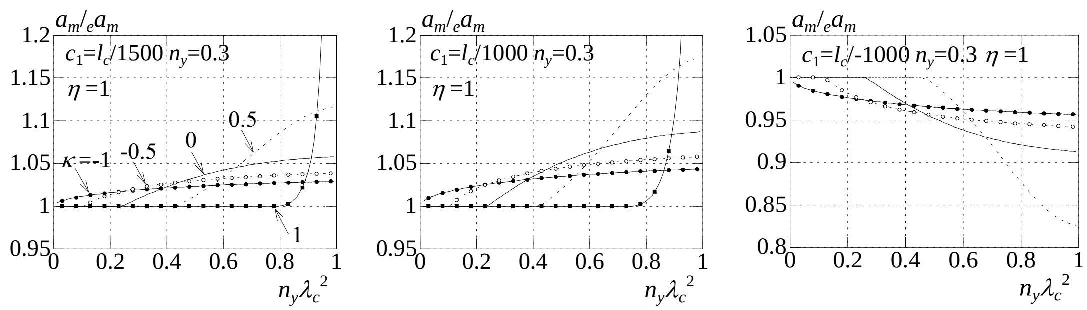
In case that κ is equal to 0, $n_y\lambda_c^2$ is about 0.5 and the value of $a_m/_ea_m$ is about 1.04 (point S, in Figure 4(b)), which is slightly larger than case of κ = -1. In the case of κ = 1 and $a_m/_ea_m$ > 1, the value of $a_m/_ea_m$ tends to rapidly increase as $n_y\lambda_c^2$ increases.

3.2.2 *Influence of end moment ratio κ*

Figure 5 shows the effect of the end moment ratio on the value of $a_m/_ea_m$. Figure 5 (a), (b) and (c) show the cases where the initial deflection c_1 is l_c/1500, l_c/1000, and -l_c/1000, respectively. In these figures, the cases of κ = -1, -0.5, 0, 0.5 and 1 are indicated by ●, ○, solid line, dotted line and ■ marks.

According to Figures 5(a) and (b), when κ is equal to -1, the value of $a_m/_ea_m$ is larger than 1 from the point where the value of $n_y\lambda_c^2$ is close to 0. At the point where the value of $n_y\lambda_c^2$ is close to unity, the values of $a_m/_ea_m$ are about 1.03 and 1.05, respectively. The larger the end moment ratio, the larger the value of $a_m/_ea_m$ when the value of $n_y\lambda_c^2$ is close to 1.

As shown in Figure 5(c), when the initial deflection c_1 is negative, the value of $a_m/_ea_m$ becomes less than or equal to 1. The case of $\kappa = 1$, where the value of $a_m/_ea_m$ is negative, is not shown in Figure 5(c) and the absolute value of $a_m/_ea_m$ is equal to the case of $\kappa = 1$ in Figure 5 (b).



(a) $c_1 = l_c/1500$ (b) $c_1 = l_c/1000$ (c) $c_1 = -l_c/1000$

Figure 5. $a_m/_ea_m$-$n_y\lambda_c^2$ relations (influence of end bending moment ratio κ).

4 CONCLUSIONS

The conclusions derived from this study are as follows:

1) Moment amplification factor a_m is related to the end moment ratio κ, $n_y\lambda_c^2$ (n_y : axial load ratio, λ_c: normalized slenderness ratio) and Nc_j/M_1. The value of Nc_j/M_1 is defined by the non-dimensional values n_y, c_j and η.

2) As to the effect of initial deflection, regardless of the value of the end moment ratio κ, as the initial deflection c_1 increases, the moment amplification factor a_m becomes large. In the case of $c_1 = l_c/1500$ given as management tolerance of steel columns, the ratio of $a_m/_ea_m$ is at most 1.04 when $n_y\lambda_c^2$ is 0.5, and the initial deflection c_1 has little influence on the value of $a_m/_ea_m$.

3) As to the effect of the end moment ratio κ, as the value of $n_y\lambda_c^2$ is close to unity when the value of κ is -1, the values of $a_m/_ea_m$ are about 1.03 and 1.05, respectively. The larger the end bending moment ratio, the larger the value of $a_m/_ea_m$ when the value of $n_y\lambda_c^2$ is close to 1.

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