Resilient Structures and Sustainable Construction
Edited by Pellicer, E., Adam, J. M., Yepes, V., Singh, A., and Yazdani, S.

ISBN: 978-0-9960437-4-8

RELATIONSHIP BETWEEN BATTENED BUILT-UP COLUMNS AND FRAMES ABOUT BUCKLING – BUCKLING STRENGTH BY BLEICH

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The objective of this study is to examine the relationship between battened built-up columns and frames about buckling. The buckling strengths, buckling modes and bending moment of one bay - *n* stories frame with rigid beams are calculated by the energy method. The results are compared with those of obtained by the Bleich's formula and correct solutions. Although the buckling strengths by Bleich agree fairly well with the correct values, it is shown that the strengths obtained by Bleich formula are neither the upper bound nor the lower bound. And buckling strengths calculated by the energy method estimate the correct strength within 5% error when the value of stories *n* is more than 2. Moreover, Bleich's moment distributions correspond to those of obtained by the energy method.

Keywords: Built-up compression member, Flexural buckling, Energy method, Buckling equation, Moment distribution, Bleich's formula.

1 INTRODUCTION

In this paper, battened built-up columns which have rigid batten plates are targeted, and the correct solution is obtained as buckling strengths of a frame with considering shrinkage and elongation of the column. The relationships between the buckling strengths of the built-up compression member obtained by using the Bleich's formula, the buckling strengths of the frame obtained by the energy method considering the shrinkage and elongation of the columns and the correct buckling strengths (Timoshenko 1961) were examined.

Bleich calculated the buckling strengths of the built-up compression member based on the conservation law of energy. Although it is widely known that the Rayleigh-Ritz method gives the upper bound of the buckling strengths, little is known about the relationship between the buckling strengths by Bleich and the correct buckling strengths. In this paper, the bending moment diagram of the column is calculated by the energy method, and the influence of the governing parameter is considered from the viewpoint of buckling of the frame.

2 ANALYTICAL WORK

2.1 Problem Setting and Analytical Method

The built-up columns to be covered in this study is the battened built-up columns shown in Figure 1. The analytical model is *n* stories frame which has rigid beams and the fixed column-base shown in Figure 2. This model corresponds to a half of the pinned end built-up column shown in Figure 1. The built-up column has a batten plate at the mid-point of the column, and whose number of sections divided by plates is $2n$. There are cases where the number of interval divided by the batten plate is an odd number, but in this paper the case of an even number is targeted. In Figure 2, E is the Young's modulus, I_c is the second moment of inertia of the column, A_c is the cross-sectional area of the column, h is the height of the frame, c is the story height, and b is the span length.

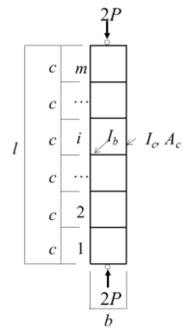


Figure 1. Battened built-up columns.

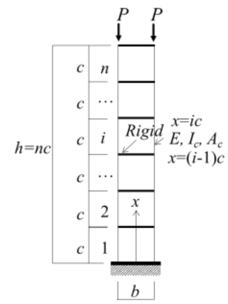


Figure 2. Analytical model.

2.2 Buckling Strengths of Built-up Columns by Bleich

The buckling strengths $2P_{cr}$ of the built-up columns considering the shear deformation is expressed by Bleich (1952).

$$2P_{cr} = \frac{\pi^2 EI_g}{l^2} \frac{1}{1 + \dfrac{\pi^2 EI_g}{l^2}\left(\dfrac{cb}{12EI_b} + \dfrac{c^2}{24EI_c}\right)} \tag{1}$$

In Eq. (1), E is the Young's modulus, l_a, l_b and l_c are the second moment of inertia of the built-up columns, chord member and plate respectively, l is the member length, c is the distance between the plates, and b is the plate length. The second moment of inertia I_g of the built-up column can be calculated by Eq. (2), where A_c is the cross-sectional area of the chord member.

$$I_g = 2\left(\frac{b}{2}\right)^2 A_c + 2I_c \tag{2}$$

Considering the correspondence of Eq. (1) to the analytical model of Figure 2 derives the relation $l = 2h = 2nc$, and substituting this relation and Eq. (2) into Eq. (1) and arranging it gives Eq. (3).

$$P_{cr} = \frac{\pi^2 EI_c}{c^2} \cdot \frac{1 + \dfrac{4I_c}{A_c b^2}}{\left\{4n^2 + \dfrac{\pi^2}{12}\left(\dfrac{2b}{c} \cdot \dfrac{I_c}{I_b} + 1\right)\right\} \cdot \dfrac{4I_c}{A_c b^2} + \dfrac{\pi^2}{12}\left(\dfrac{2b}{c} \cdot \dfrac{I_c}{I_b} + 1\right)} \tag{3}$$

Assuming $I_b = \infty$ in Eq. (3) and normalizing the buckling strengths in Eq. (3) by P_e in Eq. (4), the non-dimensional buckling strengths p^* of the built-up columns by Bleich is given by Eq.

(5). P_e in Eq. (4) is the buckling strengths of columns whose length is c and fixed at both ends with movement of nodes. In this paper, Eq. (5) is called "buckling strengths by Bleich".

$$P_e \equiv \frac{\pi^2 EI_c}{c^2} \tag{4}$$

$$p^* \equiv \frac{P_{cr}}{P_e} = \frac{1+\dfrac{4I_c}{A_c b^2}}{\left(4n^2+\dfrac{\pi^2}{12}\right)\cdot\dfrac{4I_c}{A_c b^2}+\dfrac{\pi^2}{12}} \tag{5}$$

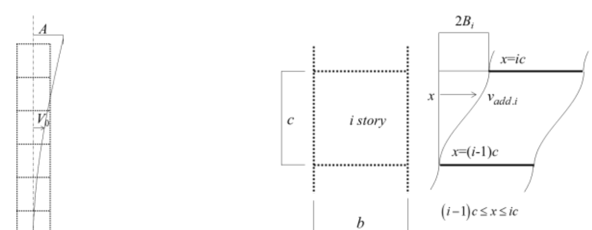
From Eq. (5), the non-dimensional parameters that dominates the non-dimensional buckling strengths p^* are the value of $4I_c/A_cb^2$ and the number of stories n. The non-dimensional buckling strengths P_g/P_e in the case of not considering the shear deformation in Eq. (1) is obtained by Eq. (6). P_g in Eq. (6) is the buckling strengths of the built-up compression member without considering the shear deformation.

$$\frac{P_g}{P_e} \equiv \frac{\dfrac{1}{2}\cdot\dfrac{\pi^2 EI_g}{(2h)^2}}{\dfrac{\pi^2 EI_c}{c^2}} = \frac{1}{4n^2}\cdot\left(1+\frac{1}{\dfrac{4I_c}{A_c b^2}}\right) \tag{6}$$

2.3 Calculation of Buckling Strengths Using the Energy Method

2.3.1 *Assumption of column deflection*

In this section, the buckling strengths of the frame considering shrinkage of the column is calculated by using the energy method. The analytical model is shown in Figure 3.



As i is an odd number *As i is an even number*

$v_0 = A\left(1-\cos\dfrac{\pi}{2h}x\right)$ $v_{add.i} = B_i\left(1-\cos\dfrac{\pi}{c}x\right)$ $v_{add.i} = B_i\left(1+\cos\dfrac{\pi}{c}x\right)$

(a)Deflection as a column. (b) Deflection as a frame.

Figure 3. Analytical model.

The deflection of the column is expressed as the sum of the deflections: v_0 as the built-up column and the additional deflection v_{add} as the frame (refer to Figure 3(a) and 3(b)). $v_{add.i}$ is an additional deflection with respect to the column-base of the i story, and the lower right suffix " i " means that it is related to the i story. $v_{add.i}$ on the left side of Figure 3 relates to an odd number story, and $v_{add.i}$ on the right side relates to an even number story.

2.3.2 Total potential energy (Π) and buckling equation

Various amounts in the *i* story are calculated, the total strain energy *U* and the potential energy *V* of the external force are obtained. By adding the strain energy and the potential energy of the external force, the total potential energy (Π) is given by Eq. (7).

$$\Pi = U + V = \frac{1}{2} EI_g \int_0^{nc} v_0''^2 dx + \frac{1}{2} EI_c \sum_{i=1}^{n} \int_{(i-1)c}^{ic} 2v_{add.i}''^2 dx$$
$$+ \frac{1}{2} EI_c \sum_{i=1}^{n} \int_{(i-1)c}^{ic} 4v_0'' v_{add.i}'' dx - p \int_0^{nc} v_0'^2 dx - P \sum_{i=1}^{n} \int_{(i-1)c}^{ic} v_{add.i}'^2 dx - P \sum_{i=1}^{n} \int_{(i-1)c}^{ic} 2v_0' v_{add.i}' dx \tag{7}$$

Buckling equation is derived based on the principle of minimum potential energy. The condition for minimizing the total potential energy Π is given by the Eq. (8).

$$\frac{\partial \Pi}{\partial A} = \frac{\partial \Pi}{\partial B} = 0 \qquad (i = 1, 2, \ldots, n) \tag{8}$$

From Eq. (8), the buckling equation is obtained as the determinant consisting of the elements of $(n+1) \times (n+1)$ which is equal 0. The element can be represented by $p^*(\equiv P/P_e)$, P_g/P_e . P_g/P_e is given by Eq. (6).

2.4 Buckling Strengths of Frame Considering Axial Shortening of Columns

The following equation is the buckling strengths *P* of a frame with rigid beams, which is obtained by the buckling slope deflection method and the shrinkage and elongation of the column is further considered. The symbols defined in this paper are used in the following equation.

$$P = \frac{z^2 EI_c}{c^2} \tag{9}$$

z in the numerator on the right side of the above equation is calculated from the following equation. In this paper, we call the buckling strengths calculated from Eq. (9) and Eq. (10) the correct buckling strengths.

$$\frac{4I_c}{A_c b^2} = \frac{1 - \cos(\pi/m)}{\cos(\pi/m) - \cos z} \cdot \frac{\sin z}{z} \tag{10}$$

In Eq. (10), *m* is the number of stories in the frame with pin supported at both ends, and there is a relationship of $m = 2n$ and *n* is the number of stories in this paper. Also, when the value of $4I_c/A_c b^2$ is 0 or ∞, the value of *z* is π, $\pi/m = \pi/2n$, the value of buckling strengths is $\pi^2 EI_c/c^2$ and $\pi^2 EI_c/4n^2c^2$, and the value of the non-dimensional buckling strengths p^* is 1 and $1/4n^2$.

3 RESULTS AND DISCUSSION

3.1 Analytical Variable

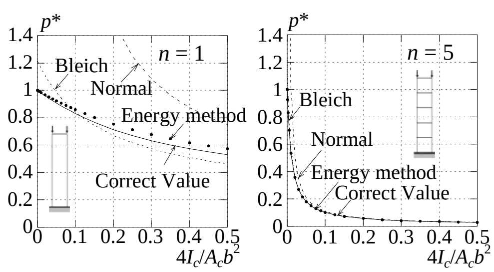
The parameter that dominates the non-dimensional buckling strengths is the value of $4I_c/A_c b^2$ and the number of stories *n* according to Eq. (5) and Eq. (10). As the analytical parameters, calculation was performed by setting the number of stories *n* to 1, 2, 3, 5, 10, and changing the value of $4I_c/A_c b^2$ from 0 to 1. In the case of $4I_c/A_c b^2 = 0$, the span *b* is large, so that the axial force of the column is small and buckling of a normal frame is considered without considering shrinkage and elongation of the column. And when $4I_c/A_cb^2 = \infty$, the built-up compression member buckles as one column. It is considered the upper limit of $4I_c/A_cb^2$ value is about 0.5 for the battened built-up columns according to the past studies (Atushi *et al.* 2015).

3.2 Non-dimensional Buckling Strengths p^* - $4I_c/A_cb^2$ Relationship

In Figure 4 shows the relationships between $4I_c/A_cb^2$ and the non-dimensional buckling strengths p^*. The relationships obtained by using the Eq. (9) and Eq. (10) (Described as "Correct Value" in Figure.), the energy method and the Bleich's buckling formula expressed by Eq. (5) are indicated by solid lines, black circles and dotted lines, respectively. In addition, the broken line shows the strengths when the built-up column buckles in a usual manner without considering the influence of the shear deformation of the battened built-up columns (Described as "Normal" in FIG).

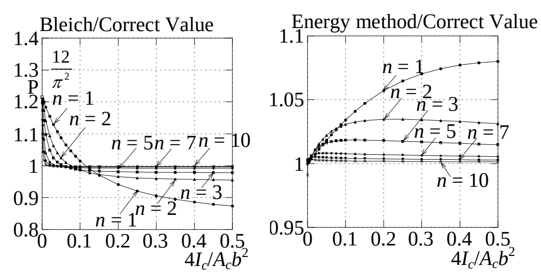
When the number of stories n equals 1, there is some difference between the correct value and the strengths by the energy method, but the difference decreases as the value of n increases. In addition, according to Eq. (5) as the built-up compression member, when the value of $4I_c/A_cb^2$ is small, the non-dimensional buckling strengths p^* is evaluated larger than the correct value. As the value of $4I_c/A_cb^2$ increases, the buckling strength is evaluated as smaller than the correct value.

Figure 5 shows a comparison with the correct value. Figures 5(a) and (b) show the results by Bleich's formula and by the energy method. According to Figure 5(a), when $4I_c/A_cb^2$ equals 0, the value of Bleich /correct is $12/\pi^2$, and the value exceeds 1 when $4I_c/A_cb^2$ is small. Also, it becomes smaller than 1 when the value of $4I_c/A_cb^2$ increases. It means that the strengths by Bleich are neither the upper bound nor the lower bound. When the value of n becomes large and the value of $4I_c/A_cb^2$ becomes about 0.1 or more, it is observed that the buckling strengths by Bleich agree fairly well with the correct value.



(a) $n = 1$ (b) $n = 5$

Figure 4. Comparison of buckling strengths.



(a) Bleich's formula. (b) Energy method.

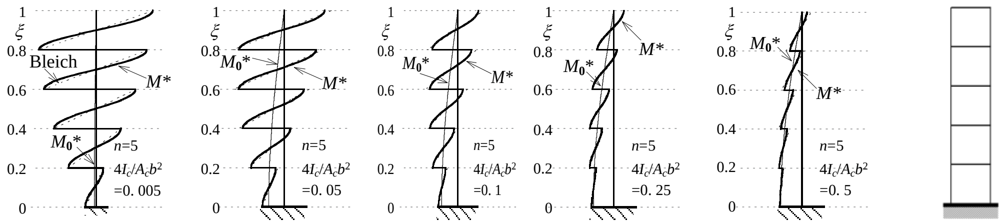
Figure 5. Comparison with the correct value.

According to Figure 5(b), buckling strengths calculated by the energy method estimate the correct value within 5% when the value of stories n is more than 2. As the value of n increases, the buckling strengths by the energy method agree the correct value. As is well known, the buckling strength by the energy method gives the upper bound.

3.3 Bending Moment Distribution

Figure 6 shows the bending moment distribution for n equals 5. The value of $4I_c/A_cb^2$ was calculated as 0.005, 0.05, 0.1, 0.25, and 0.50. The bold solid line, thin solid line and dotted line in the figure are the bending moment M^* of the column, the bending moment M_0^* calculated as a single column for the whole frame, and the bending moment assumed by Bleich. The bending moment of the column base is taken as unit 1 (The horizontal axis is set to 1/2 in size only in

Figure 6(a)). According to these figures, when $4I_c/A_cb^2$ = 0.005, the inflection point is located between column base and top in all stories. As the value of $4I_c/A_cb^2$ increases, it is observed that the curvature is bent into a single curvature in order from the lower story to the upper story. Also, it is observed that Bleich's moment distributions correspond to those of obtained by the energy method.



(a) $4I_c/A_cb^2$=0.005 (b) 0.05 (c) 0.1 (d) 0.25 (e) 0.5 Five-stories frame

Figure 6. Bending moment distribution.

4 CONCLUSIONS

In this paper, with the aim of considering the relationship between buckling of battened built-up columns and buckling of frame, the buckling strengths of battened built-up columns by Bleich, the buckling strengths calculated by the energy method considering approximately the shrinkage and elongation of columns and the correct buckling strengths presented by Timoshenko are calculated. The conclusions derived from this study are as follows:

1) Governing parameters of the problem are the number of story n and the value of $4I_c/A_cb^2$.
2) Although the buckling strengths by Bleich agree fairly well with correct values, the strengths by Bleich are neither the upper bound nor the lower bound.
3) Buckling strengths calculated by the energy method estimate the correct strength within 5% when the value of stories n is more than 2. As the value of n increases, the buckling strength by the energy method well matches the correct value. As is well known, the buckling strength by the energy method gives the upper bound.
4) Bleich's moment distributions correspond to those of obtained by the energy method.

Acknowledgements

This study was supported by the Grant-in-Aid for Scientific Research of Japan Society for the Promotion of Science.

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