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CALCULATION OF THE NATURAL VIBRATIONAL FREQUENCY OF THE PULSE-LOADED EMBEDDED FOUNDATION

ALEKSEY KOLESNIKOV[1], VLADIMIR POPOV[2], and TATIANA KOSTIUK[1]

[1]*Novosibirsk State University of Architecture and Civil Engineering (SIBSTRIN), Novosibirsk, Russia*
[2]*Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, Novosibirsk, Russia*

Applicability of formulas obtained within the framework of wave model and Russian design code SP 26.13330.2012 were considered to determine the dynamic stiffness in respect to vertical and horizontal vibrations of shallow and pile embedded foundations. The target is to calculate the system's varying natural vibrational frequencies regarding the deepening. Obtained results are compared with experimental data of a series of pulse dynamic loadings under half-full-scale conditions on foundation models. Values of transversal wave velocities are found directly on test section from test results. It is found that deepening results in natural vibrational frequencies growth, both in case of pile and shallow foundations. Full deepening of more than 1.6 times increases frequencies at horizontal vibrations, and up to 1.4 times at vertical vibrations; the effect is stronger for shallow foundations. Benefit of the results obtained by wave model against the SP 26.13330.2012 method is demonstrated, as well as the good agreement between calculation and experimental results, which permits finding reliable amplitude-frequency characteristics of the foundations. Results obtained in accordance with SP 26.13330.2012 show lower values of natural vibrational frequencies and do not completely regard the deepening effect. Maximal discrepancy with experimental data is 33% for vertical vibrations on shallow and pile foundations. In case of horizontal vibrations, the maximal discrepancy is from 20% for shallow foundation to 27% for shallow and pile foundation.

Keywords: Machine foundation, Wave model, Eigenfrequency of the foundation.

1 GENERAL APPEARANCE

Today, there are special requirements for the accuracy of the results of the foundation vibrations calculation (Zabylin 1983, Koloushek 1965). Thus, the models in use have to give a fuller picture about the dynamic characteristics of the "embedded foundation-base" system (Baranov 1967, Novak and El Sharnouby, 1983, Kolesnikov and Popov 2009, Kolesnikov and Popov 2010). The increased stiffness of the foundation, due to the interaction with the ambient medium of the embedded side edges, results in the increased frequencies of the natural vibrations, which is proven experimentally; the experiments were carried out in different conditions (Koloushek 1965, Kolesnikov and Popov 2010). Hence, the effect of the dense contact of the foundation side surface and ambient soil considered for the calculation and design of the machine foundations permits determining more accurately the values of the amplitude-frequency characteristics of the whole system vibrations (Baranov 1967).

At the same time, when calculating or investigating the characteristics of shallow and pile embedded foundations, there are some discrepancies with the experimental results (Kolesnikov and Popov 2010). It is perhaps necessary to take into account the foundation geometry and other individual characteristics in order to increase the calculation accuracy, but it requires detailed analysis (Nuzhdin and Kolesnikov 2004, Kolesnikov and Popov 2017). The present work addresses to the engineering calculations involving the formulas from Baranov (1967), Nuzhdin and Kolesnikov (2004) to determine the natural frequencies at the vertical and horizontal vibrations of the foundations, with due regard to their deep-grid position and comparison of the results with the data obtained experimentally in a test landfill.

2 INVESTIGATION TECHNIQUE

To evaluate the effect of the deepening on the natural frequencies at the vertical and horizontal vibrations of the foundations, the experimental investigations were carried out in accordance with the schematic in Figure 1.

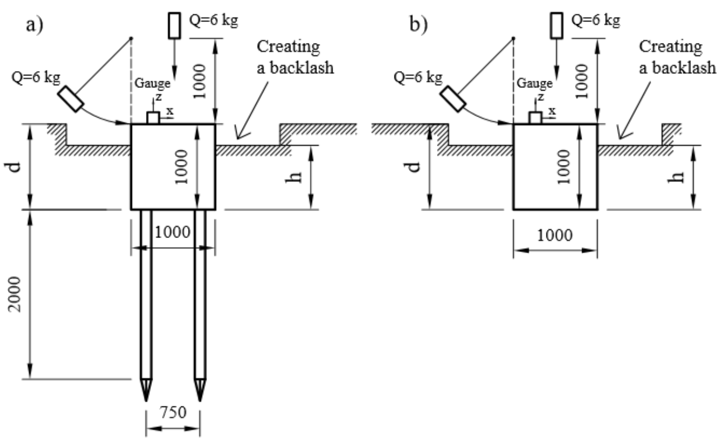

Figure 1. Schematic of tested foundations F-1 (a) and F-2 (b):
h – the height of backfill, *d* – the depth of foundation.

Two specially made foundations were used; one of them is the pile foundation (F-1), and the other is the shallow foundation (F-2). The foundation F-1 was tested as a monolithic reinforced concrete pile cap, its overall sizes are 1.0×1.0×1.0 m; it is based on four rigidly fixed piles with the diameter of 114 mm (the working length 2 m) and is made from metal tubes with the wall thickness of 6 mm. The distance between the pile axes was 6.5 diameters (750 mm), which excluded their mutual influence in the cluster (Kolesnikov and Popov 2017). The monolithic shallow foundation F-2 had the same sizes 1.0×1.0×1.0 m. Thoroughly leveled pit walls served as a concrete form for the test foundations. To the depth of 9.3 m, the soil of the test landfill was the loess slightly wet hard sand clay, its specific weight 17.0 kN/m^3, the deformation modulus E = 14 MPa; the bed layer is semi-solid clay loam. There was no subterranean water in the landfill.
The series of dynamic impulse loadings (20 times) were applied for different versions of deepening, the technique was the same. The impulse loading was applied with the aid of a steel block shaped as a parallelepiped of 6 kg. To produce the vertical loading, the block fell freely onto the tested foundation surface from the height of 1.0 m. To produce the horizontal loading, a pendulum was used; it was hung of a string above the foundation side edge at a height of 1.0 m and bumped the upper foundation edge. At the complete foundation deepening, some soil near

the impact point was removed to exclude interferences. To determine the vibration natural frequencies, the special automatic vibrometer was used. It permitted registering the vibrations and simultaneously processing the obtained information. The measurement data for each foundation F-1 and F-2 were compared with the engineering calculation results (the analytic relations from Baranov 1967, Nuzhdin and Kolesnikov 2004 and SP 26.13330.2012). The natural frequencies of the foundation vibrations, according to the wave model, λ_z at the vertical and λ_x at the horizontal vibrations of the foundations are defined in Eq. (1):

$$\lambda_z = \sqrt{K_z/m},\ \lambda_x = \sqrt{K_x/m}, \tag{1}$$

where K_z, K_x are the foundation vertical and horizontal stiffness, respectively, m is the foundation mass.

The approach to the determination of the embedded foundation dynamic stiffness based on wave models suggests summing-up the values found separately for the side surface, bottom and piles if there are any (Zabylin 1983, Kolesnikov and Popov 2009). The vertical (z) and horizontal (x) stiffness of the embedded test foundations are found as follows:

For the pile foundations: summing-up the stiffness on the side surface of the pile cap K_1 and stiffnesses K_2 of each pile from N in the base is calculated as in Eq (2):

$$K_z = K_{z1} + \sum_{i=1}^{N} K_{z2_i},\ \ K_x = K_{x1} + \sum_{i=1}^{N} K_{x2_i}, \tag{2}$$

For the shallow foundations: summing-up the stiffnesses on the side surface of the deep-grid foundation K_1 and under its bottom K_3 can be found in Eq. (3).

$$K_z = K_{z1} + K_{z3},\ K_x = K_{x1} + K_{x3}. \tag{3}$$

The vertical stiffness by the side surface of the pile cap or foundation is found in Eq. (4) as follows:

$$K_{z1} = V_{s0}^2 \rho_0 S_{w1} h, \tag{4}$$

where ρ_0 is the backfill soil density and V_{s0} is the speed of transversal waves in the foundation (pile cap) backfill, h is the backfill height, S_{w1} is the coefficient detected by the dimensionless vibration frequency $a_0 = r\omega/V_{s0}$ and geometrical shape in the embedded foundation (pile cap) plane, ω is the circular vibrational frequency, r is the characteristic size, $r = r_0$ for the circular in plane foundation with the radius of r_0, $r = \sqrt{ab/\pi}$ for the body rectangular in plane, with the sides a and b (Baranov 1967, Zabylin 1983, Nuzhdin and Kolesnikov 2004).

The vertical stiffness of a single pile is detected by the formula shown in Eq. (5).

$$K_{z2} = f_{19.1} E_p A_p / r_p\,, \tag{5}$$

where E_p is the pile material elasticity modulus, A_p is the pile cross section area, r_p is the circular pile radius, or $r_p = c/\sqrt{\pi}$ in the case of a rectangular pile with the side c, $f_{19.1}$ is the coefficient found in accordance with Novak and Sharnouby (1983).

The vertical stiffness under the foundation bottom is found using the formula in Eq. (6).

$$K_{z3} = V_s^2 \rho r F_{1z}, \tag{6}$$

where ρ is the soil density and V_s is the transversal wave speed in the soil under the foundation bottom, F_{1z} is the coefficient detected in accordance with Zabylin (1983).

Horizontal stiffness by the side surface of the pile cap or foundation is found by the formula in Eq. (7).

$$K_{x1} = V_{s0}^2 \rho_0 S_{u1} h, \tag{7}$$

where S_{u1} is the coefficient in accordance with (Baranov 1967, Zabylin 1983, Nuzhdin and Kolesnikov 2004) related to the dimensionless vibration frequency a_0 and geometrical shape of the embedded foundation (pile cap).

The horizontal stiffness of the single pile is found by the formula in Eq. (8).

$$K_{x2} = f_{11.1} E_p I_p / r_p^3, \tag{8}$$

where I_p is the pile cross section inertia moment, $f_{11.1}$ is the coefficient detected in accordance with Chowdhury and Dasgupta (2008).

The horizontal stiffness under the foundation bottom is found by the formula in Eq. (9).

$$K_{x3} = V_s^2 \rho r F_{1x}. \tag{9}$$

3 INVESTIGATION RESULTS

Tables 1 and 2, respectively, present the measurement and calculation results for the natural vibrational frequencies of the foundations F-1 and F-2. The natural vibrational frequencies were registered for each tested foundation along with the calculations by the formulas in Eq. (2), (3) for different levels of their relative deepening *h*/*d*=1; 0,75; 0,5; 0,25; 0.

Table 1. Natural frequencies of foundation F-1.

Item No	**Deepening, *h/d***	**Measured frequency, Hz**		**Calculated frequency, Hz**					
				SP 26.13330.2012		**Baranov (1967)**		**Nuzhdin and Kolesnikov (2004)**	
		Hor.	Vert.	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
1.	1.00	42.47	54.54	31.04	36.17	39.97	56.59	41.33	59.01
2.	0.75	41.43	52.25	30.09	35.46	36.96	54.33	39.57	56.23
3.	0.50	38.71	50.00	28.60	34.75	35.92	51.96	37.73	53.29
4.	0.25	33.85	47.61	26.48	33.27	34.85	49.49	35.79	50.19
5.	0.00	30.03	45.40	24.17	31.72	33.75	46.88	33.75	46.88

Table 2. Natural frequencies of the foundation F-2.

Item No	**Deepening, *h/d***	**Measured frequency, Hz**		**Calculated frequency, Hz**					
				SP 26.13330.2012		**Baranov (1967)**		**Nuzhdin and Kolesnikov (2004)**	
		Hor.	Vert.	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
1.	1.00	36.10	45.76	28.83	30.30	34.68	44.74	38.27	47.79
2.	0.75	32.25	42.30	27.95	29.70	32.08	41.83	35.01	44.29
3.	0.50	28.80	39.50	26.56	29.10	29.25	38.70	31.41	40.49
4.	0.25	25.45	35.25	24.60	27.85	26.11	35.30	27.34	36.29
5.	0.00	22.30	31.15	22.45	26.56	22.55	31.54	22.55	31.54

The calculations were carried out in accordance with Baranov (1967) for the foundation circular in the plane and Nuzhdin and Kolesnikov (2004) for the body rectangular in the plane and in accordance with SP 26.13330.2012. Their mutual influence was not regarded when determining the total pile stiffness in the base (Kolesnikov and Popov 2017).

It should be noted that the calculations involved the values of the transversal wave speeds V_s, detected directly in the test landfill by the method described in Chowdhury and Dasgupta (2008). The speed values for the test landfill soil are detected as V_{s1} = 146 m/s (the vertical loading) and V_{s2} = 105 m/s (the horizontal loading). Different values obtained for different impact direction prove the anisotropy of elastic wave speeds in the soil (Petrashen' 1980).

It follows from the results of the experiments carried out in the test landfill that the increased depth leads to the increased natural vibrational frequencies, both for the pile and shallow foundations. The dependence of the frequency on the depth is almost linear (see Figure 2).

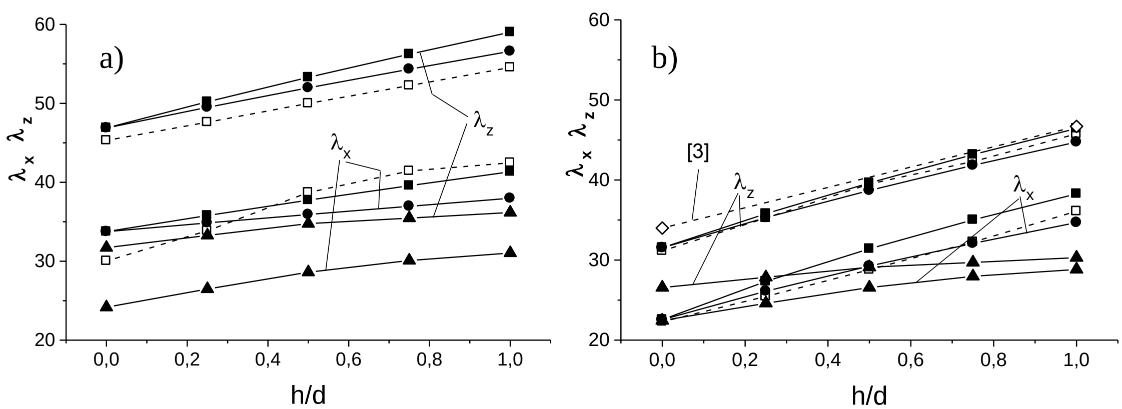

Figure 2. Correlation of the resonant frequencies λ_x, λ_z from the relative depth of foundation h/d: a. F-1, b. F-2. Dashed lines - measurements results of the authors (□) and Koloushek (1965)(◊), solid lines - the calculations results according to Baranov (1967) (•), Nuzhdin and Kolesnikov (2004) (■) and SP 26.13330.2012 (▲).

The complete deepening stimulates the 1.5 times growth of the frequency at the horizontal vibrations for the pile foundation, for the shallow foundation the growth exceeds 1.6 times, at the vertical vibrations for the pile foundation the growth is 1.3 times, and for the shallow foundation, it is 1.4 times. Note that the deepening influences more considerably in the case of the shallow foundations on the variation of the natural vibrational frequencies, than in the pile foundation case. Figure 2b presents the data from Koloushek (1965) as the vindication of the obtained results for the foundation of the same sizes and with similar soil properties. The results presented in Tables 1 and 2, obtained in accordance with SP 26.13330.2012, illustrate much lower values of the calculated natural vibrational frequencies, than the ones detected in the test measurements.

The Tables and Figure 2 show the results obtained in the engineering calculations involving the formulas (2), (3) for the detection of the stiffnesses of the embedded pile and shallow foundations. Then, calculating the natural vibrational frequencies of the foundations, we find the coefficients S_{w1} and S_{u1} in accordance with Baranov (1967) for the body circular in the plane, and in accordance with Nuzhdin and Kolesnikov (2004) for the body rectangular in the plane. The presented frequency values permit stating that the calculations in the framework of the used approximations enable having a good agreement with the experimental data.

Considering the results obtained with S_{w1} and S_{u1}, found by the formulas in accordance with Baranov (1967) for the body circular in the plane, the discrepancy with the experimental data at the horizontal vibrations ranges from 6 % to 12.5 % for the pile foundations, and up to 4 % for the shallow foundations. At the vertical vibrations, the discrepancy with the experimental data is lower: up to 4 % for the pile foundations, about 2 % for the shallow foundations.

Evaluating the results obtained with S_{w1} and S_{u1}, calculated in accordance with Nuzhdin and Kolesnikov (2004) for the body rectangular in the plane, we have that at the horizontal vibrations, the discrepancy with the experimental data ranges from 3 % to 12.5% for the pile foundations, and 6 % for the shallow foundations. Here, the maximal discrepancy is also registered at the depth of h/d = 0 during the calculation of the base pile stiffness. At the vertical vibrations, the discrepancy with the experimental data is below 8 % for the pile foundations and 4 % for the shallow foundations.

4 CONCLUSIONS

It is found that the relations and formulas (Baranov 1967, Nuzhdin and Kolesnikov 2004) obtained within the framework of the wave model and used to calculate the dynamic stiffnesses at the vertical and horizontal vibrations of the embedded shallow and pile foundations permit obtaining more accurate results when detecting the resonance frequencies than the way proposed by SP 26.13330.2012; it is proven by the comparison of the calculation results and experimental data obtained under the half-full-scale conditions. The vibrational frequencies found by the considered formulas show better approximation to the frequencies detected experimentally than the current standard methods. Comparison of the results of the detection of the natural vibrational frequencies did not show any evident advantage of utilization of the formulas for the bodies circular or rectangular in plane, which makes it desirable to carry out further investigation of the embedded foundation under dynamic loadings, as the difference between their sides is essential, as well the analysis of the amplitude characteristics of the system.

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