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AN ENGINEERING RELIABILITY ANALYSIS METHOD BASED ON ADAPTIVE METAMODELS

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In practical engineering problems, numerical analyses using the finite element (FE) method or other methods are generally required to evaluate system responses including stresses and deformations. For problems involving expensive FE analyses, it is not efficient or straightforward to directly apply conventional sampling-based or gradient-based reliability analysis approaches. To reduce computational efforts, it is useful to develop efficient and accurate metamodeling techniques to replace the original FE analyses. In this work, an adaptive metamodeling technique and a First-Order Reliability Method (FORM) were integrated. In each adaptive iteration, a compactly supported radial basis function (RBF) was adopted and a metamodel was created to explicitly express a performance function. An alternate FORM was implemented to calculate reliability index of the current iteration. Based on the design point, additional samples were generated and added to the existing sample points to re-generate the metamodel. The accuracy of the RBF metamodel could be improved in the neighborhood of the design point at each iteration. This procedure continued until the convergence of the reliability analysis results was achieved. A numerical example was studied. The proposed adaptive approach worked well and reliability analysis results were found with a reasonable number of iterations.

Keywords: Finite element (FE), First-order reliability method (FORM), Radial basis functions (RBFs).

1 INTRODUCTION

Reliability analysis of civil engineering problems has attracted considerable attention in the last few decades (Hohenbichler *et al.* 1987, Ang and Tang 1975, Li and Low 2010, Au and Wang 2014). Commonly used reliability analysis methods include first-order and second-order reliability methods (FORM/SORM) (Hohenbichler *et al.* 1987, Low and Tang 2007). In addition, various sampling methods are also available, e.g., Monte Carlo simulations (MCS) (Rubinstein 1981) and subset simulations (Au and Wang 2014). For engineering practice, a numerical analysis method such as the FE analysis is routinely used to evaluate system responses. Therefore, the numerical analysis codes needs to be integrated with MCS or FORM/SORM for practical applications.

For engineering applications that require expensive response analyses, it is useful to develop approximate models to replace the implicit performance functions. This is referred to as a metamodel and the system responses are written explicitly in terms of input variables (Bai *et al.* 2012). A single quadratic function is commonly used to replace an implicit function in a metamodel (Faravelli 1989). This is called response surface method (RSM), which can be used to solve a variety of engineering problems, including design optimization, reliability analysis, as well as reliability-based design optimization (Youn and Choi 2004, Lü and Low 2011, Lü *et al.* 2011). To improve the model accuracy for highly nonlinear functions, different techniques have been studied, including support vector machines (Zhao *et al.* 2014), radial basis functions (Wu 1995, Fang and Horstemeyer 2006, Yin *et al.* 2016), and high dimensional model representation (Tunga and Demiralp 2005). FORM/SORM and MCS can be employed to calculate the reliability index, once an explicit metamodel function becomes available.

It is useful to investigate accurate and efficient approximation models that are applicable to complex engineering problems. A new reliability analysis method is presented in this paper that integrates FORM and an adaptive metamodeling technique. The basic concept of an adaptive metamodel is that the model accuracy can be improved in each reliability analysis iteration, when additional sample points are introduced. The metamodels are based on augmented RBFs, and the reliability index is found based on an alternate FORM in literature. In the next section, the proposed method is first introduced. Finally, a numerical example and some concluding remarks are presented.

2 THE RELIABILITY ANALYSIS METHOD

The proposed method based on combined FORM and an adaptive RBF technique is introduced in this section.

2.1 Reliability Analysis

The failure probability, P_F, is based on the following integration (Madsen *et al.* 1986):

$$P_F \equiv P(g(\boldsymbol{x}) \leq 0) = \int_{g(\boldsymbol{x})\leq 0} p_X(\boldsymbol{x}) d\boldsymbol{x} \tag{1}$$

where $\mathbf{x}$ represents all random variables in a vector form and $g(\mathbf{x})$ is a performance function. In Eq. (1), the failure probability P_F varies between 0.0 and 1.0.

2.2 An RBF-Metamodeling Technique

A RBF metamodel can be created using n sample points in Eq. (2) (Fang and Horstemeyer 2006):

$$\tilde{g}(\mathbf{x}) = \sum_{i=1}^{n} \boldsymbol{\lambda}_i \boldsymbol{\phi}(\|\mathbf{x} - \mathbf{x}_i\|) \tag{2}$$

where ϕ is a basis function and λ_i is the weighted coefficient. In this work, an augmented RBF metamodel combining the compactly supported basis function $\phi_{2,0}$ (Wu 1995) and a polynomial function was used (Fang and Wang 2008).

2.3 FORM

For correlated normal random variables, the reliability index is defined as (Hasofer and Lind 1974):

$$\beta = \min_{\mathbf{x}\in F} \sqrt{(\mathbf{x}\text{-}\boldsymbol{\mu})^{\mathrm{T}} \mathbf{C}^{-1} (\mathbf{x}\text{-}\boldsymbol{\mu})} \tag{3}$$

An alternative interpretation of Eq. (3) was presented by Low and Tang (2007). Using an expanding ellipsoid concept in the original space of random variables, β is written as in Eq. (4):

$$\beta = \min_{\mathbf{x}\in F} \sqrt{\mathbf{n}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{n}} \tag{4}$$

where $\mathbf{n}$ is a column vector of $n_l = \frac{x_l-\mu_l^N}{\sigma_l^N} = \Phi^{-1}[F(x_l)]$. Φ^{-1} is the inverse cumulative distribution function (CDF) of a standard normal variable (Low and Tang 2007). A nonlinear optimization algorithm can be used to find β, by treating the vector $\mathbf{n}$ as design variables in optimization (Low and Tang 2007, Zhao *et al.* 2014).

2.4 An Adaptive Approach

After an initial metamodel is generated, an adaptive technique is applied. This will locally improve the RBF metamodeling accuracy. This local refinement can continue until a certain stop criterion is met. Note that the computational cost is mainly determined by the metamodel sample size, i.e., the number of FE simulations.

2.5 Overall Procedure

(i) Determine an initial sample size and generate initial sample points.
(ii) Calculate performance function values at the sample points. This step involves FE analyses or other numerical analysis methods to evaluate system responses.
(iii) Create an RBF metamodel of the performance function using all sample points.
(iv) Perform reliability analysis and find the reliability index β using the alternate FORM.
(v) Check convergence of the reliability index β. Stop the overall procedure if a convergence is achieved. Otherwise, go to the next step.
(vi) Include the current design point as an additional sample point. An additional FE analysis is required to evaluate the system responses. Go to step (iii).

3 A NUMERICAL EXAMPLE

Figure 1 shows a circular tunnel surrounded by homogeneous and isotropic rock mass (Hoek 1998, Lv *et al.* 2011, Zhao *et al.* 2014). The tunnel is subjected to a hydrostatic far field stress p_0. To support the tunnel, a uniform internal pressure p_i is applied. To be consistent with the studies found in literature, two performance functions used in this work are shown in Eq. (5) and (6):

$$\mathrm{g}_1(\mathbf{x}) = 3 - \frac{r_p}{r_0} \tag{5}$$

$$\mathrm{g}_2(\mathbf{x}) = 0.01 - \frac{u_{ip}}{r_0} \tag{6}$$

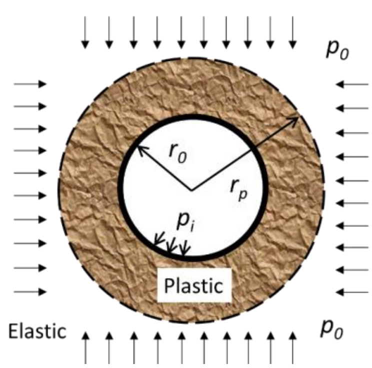

Figure 1. A circular tunnel example.

Table 1. Random variables of the tunnel example.

Variable	Distribution	Mean	Standard deviation
E	Normal	373 (Mpa)	48 (Mpa)
c	Normal	0.23 (Mpa)	0.068 (Mpa)
ϕ	Normal	22.85 (Degrees)	1.31 (Degrees)

The first performance function $g_1(\mathbf{x})$ is a criterion for tunnel plastic zone size, and the allowable size is 3. The second performance function $g_2(\mathbf{x})$ defines the inward displacement requirement. Table 1 lists random variables of the tunnel problem, i.e., E, c, and ϕ. All other variables are deterministic variables. Closed-form analytical performance functions are available and details of the functions are given in literature (Li and Low 2010). The correlation coefficient between cohesion c and friction angle ϕ was –0.5. For the first performance function, far field stresses $p_0 = 2.5$ MPa and support pressures $p_i = 0.0$ MPa. For the second performance function, far field stresses $p_0 = 2.5$ MPa and support pressures $p_i = 0.868$ MPa (Li and Low 2010). Twenty-one initial samples were generated based on the Latin hypercube sampling method. One additional sample point was generated in each adaptive RBF iteration.

For both performance functions, four adaptive iterations were required, and a total of 21+3=24 sample points were needed. The reliability analysis was also performed using FORM and closed-form analytical functions. The reliability indices and design points were calculated and the analysis results of both performance functions are listed in Table 2. For the first performance function, both methods led to β = 0.697. The reliability index β calculated using both methods was 2.504 for the second performance function. To further compare results, a global RBF metamodeling method was also used such that FORM could be applied. To obtain the global RBF metamodels, a total of 51 sample points were created based on the Latin hypercube sampling method. The global RBF method resulted in a reliability index of 0.692 and 2.510 for the two performance functions, respectively. These results are not as accurate as those obtained using the adaptive RBF approach. From this example it is seen that the adaptive RBF approach worked well and the accurate results of the reliability analysis were obtained within a few adaptive iterations.

Table 2. Reliability analysis results of the tunnel example.

		$g_1(\mathbf{x})$			$g_2(\mathbf{x})$		
		Analytical function	Adaptive RBF	Global RBF	Analytical function	Adaptive RBF	Global RBF
Far field stress	p_o (Mpa)		2.5			2.5	
Support pressure	p_i (Mpa)		0.0			0.868	
Reliability index	ß	0.697	0.697	0.692	2.504	2.504	2.510
Design point	E (Mpa)	373	373	372	259	260	260
	c (Mpa)	0.185	0.185	0.186	0.189	0.187	0.186
	ϕ (Degrees)	23.019	23.017	23.005	22.653	22.609	22.630

4 SUMMARY AND CONCLUSIONS

This work studied an effective engineering reliability analysis method, which integrated an alternate FORM and an adaptive RBF approach. The proposed method was implemented in an iterative manner. The RBF model accuracy was locally improved, due to the additional sample points identified in each analysis iteration. The new method was applied to an example problem and accurate results were obtained. A few iterations and a relatively small number of sample points were required to achieve convergence of the iterations. The proposed approach is useful for the analysis and design of complex problems that require expensive response simulations.

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