Interdependence between Structural Engineering and Construction Management
Edited by Ozevin, D., Ataei, H., Modares, M. Gurgun, A., Yazdani, S., and Singh, A.

ISBN: 978-0-9960437-6-2

STRUCTURAL OPTIMIZATION USING AN ADAPTIVE RBF APPROACH

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This paper presents an adaptive radial basis functions (RBFs) metamodeling method for design optimization of structures. Various numerical techniques have been developed and adopted in structural and multidisciplinary optimization. To evaluate responses of a structural or mechanical engineering system, finite element (FE) analyses are routinely used. An FE code shall be integrated with an optimization algorithm in a nested analysis and design of structures. Therefore, software input/output programming is required. A metamodeling method, on the contrary, expresses structural responses using an approximate function, so that the FE software is not directly coupled in the numerical optimization loop. Any optimization algorithm can be applied to find the optimal design, based on the explicit response functions. In this study, numerical examples were created and FE analyses were first performed at sample points. Subsequently, metamodels were constructed and a gradient-based optimization algorithm was applied. At the optimal point of one adaptive iteration, accuracy of the RBF metamodel was checked, and additional sample points were added to the sample pool to improve the model accuracy. The adaptive iterations continued, until the convergence of the objective function was achieved. The proposed optimization method worked well for a numerical example, and the optimal result was found within a few adaptive iterations.

Keywords: Optimal design, Finite element (FE), Adaptive metamodels, Radial basis functions (RBFs).

1 INTRODUCTION

Traditional structural and mechanical engineering system optimization relies on sensitivity analyses of system responses, since gradient-based optimization methods are widely available (Kirsch and Rozvany 1994, Arora and Wang 2005). Sensitivity-free methods, such as a genetic algorithm (GA), can also be used (Goldberg 1989); however, these involve significant computational resources, when expensive response simulations are needed, i.e., FE analyses. Approximate models have been developed to replace the implicit response functions such that the approximate functions can be directly used in a numerical optimization loop. These models are commonly referred to as a surrogate or metamodel (Jin *et al.* 2001, Fang *et al.* 2005). In literature, different types of metamodeling approaches are available, including response surface method (Bi *et al.* 2010), kriging (Jin *et al.* 2001), support vector machine (Basudhar and Missoum 2008), and RBFs (Fang *et al.* 2005).

When RBF metamodels are used, there are no errors at any sample points. Some studies showed that RBFs could be used to generate very accurate approximation functions (Fang *et al.* 2005, Fang and Wang 2008). There is a need to develop and apply accurate metamodeling methods to the design of practical civil engineering structures. An optimization method based on an adaptive RBFs is the focus of the current study. A traditional gradient-based algorithm can be used as the optimization engine.

In this paper, the structural optimization problem is first introduced. An adaptive metamodeling method based on RBFs is introduced to create explicit approximate functions of implicit structural responses, with the overall optimization steps being outlined. As a numerical example, a roof dome structure is introduced and a global buckling constraint is considered. The optimization results obtained using adaptive and traditional RBFs are compared. Finally, some concluding remarks are presented.

2 STRUCTURAL OPTIMIZATION PROBLEM

In a traditional structural optimization formulation, the objective of a design is to minimize a cost function (Eq. (1)):

$$C(\mathbf{x}) \tag{1}$$

subject to

$$\mathbf{g}(\mathbf{x}) \leq \mathbf{0} \tag{2}$$

$$\mathbf{x}^{\boldsymbol{L}} \leq \mathbf{x} \leq \mathbf{x}^{\boldsymbol{U}} \tag{3}$$

where x is a design variable vector, representing the structural size, geometry, or topology variables that need to be determined. Eq. (2) defines response or performance requirements for a given structure, including force, displacement, and buckling constraints. In Eq. (3), $\mathrm{x}^{\boldsymbol{L}}$ and $\mathrm{x}^{\boldsymbol{U}}$ are the lower and upper limits of the design variable vector x.

3 AN ADAPTIVE RBF OPTIMIATION APPROACH

In this section, the proposed adaptive RBF optimization approach and procedure are introduced. Numerical examples are discussed in the next section.

3.1 A Metamodeling Method

A metamodel of a response function $g(\mathbf{x})$ is expressed using RBFs, as (Fang *et al.* 2005, Fang and Wang 2008):

$$\tilde{g}(\mathbf{x}) = \sum_{i=1}^{n} \boldsymbol{\lambda}_{\boldsymbol{i}} \boldsymbol{\phi}(\|\mathbf{x} - \mathbf{x}_{\boldsymbol{i}}\|) \tag{4}$$

where ϕ is a basis function, λ_i is coefficient, and n is the sample size, respectively. To improve the accuracy of Eq. (4), an augmented RBF model can be used, when Eq. (4) is augmented with a linear or quadratic function (Fang *et al.* 2005, Fang and Wang 2008).

3.2 An Adaptive Technique

To progressively improve the accuracy of an RBF metamodel, an adaptive scheme is developed in this work. The key idea is to further refine the RBF model in the region of interest, i.e., the neighborhood of the optimal point of the current design iteration. The optimal point is used as an additional sample point that is included in the sample pool. The metamodel function can be re-constructed and used in the optimization in the subsequent iteration. This adaptive refinement will be repeated in each optimization iteration, until the objective function value converges.

3.3 Numerical Optimization Algorithms

Different numerical optimization algorithms are available to solve an optimization problem (Arora 2011). These include gradient-based methods, such as sequential quadratic programming (SQP) and generalized reduced gradient (GRG), and gradient-free methods such as a genetic algorithm (GA) (Goldberg 1989). In this research, the GRG solver in Excel (Microsoft 2013) was used as the optimization engine.

3.4 Overall Optimization Steps

The optimization procedure starts with the generation of initial sample points. FE analyses are conducted and RBF-metamodels are constructed using all the samples. Numerical optimization can be executed using any optimization algorithm to find an optimal design point of the current iteration. If the accuracy of a metamodel needs to be improved, the current optimal point is included in the sample pool. Therefore, an additional FE analysis should be conducted. The overall procedure of the design optimization method is highlighted as follows

(i) Generate initial samples.
(ii) Perform FE analyses at the sample points and evaluate constraint function values.
(iii) Create RBF metamodels of constraint functions using all available sample points.
(iv) Perform design optimization using the metamodel functions and obtain the current optimal design point.
(v) Perform an FE analysis at the current optimal design point to verify the accuracy of the metamodel.
(vi) Check the convergence of the objective function. If convergence is achieved, stop. If not, continue to the next step.
(vii) Add the current optimal design point to the sample pool. Go to Step (iii).

4 A NUMERICAL EXAMPLE

To study the proposed optimization method, a roof dome structure subject to a global buckling constraint was solved.

4.1 A 120-Member Roof Dome Structure

The example was a 120-member roof dome structure. This structure was studied in literature (Keveh and Talatahari 2010). In this study, the design objective of the structure was to minimize the total volume of the structure, subject to a global buckling constraint, as follows in Eq. (5-7):

$$C(\mathbf{x}) = \sum_{i=1}^{5} L_i x_i \tag{5}$$

$$\lambda_1(\mathbf{x}) \geq \lambda_1^{Limit} \tag{6}$$

$$5\ \text{in}^2 \leq x_i \leq 15\ \text{in}^2 \tag{7}$$

Due to the symmetry of the structure, five cross-sectional areas were considered as design variables (x_i, i=1,..,5). The lower and upper bound limits of the variables were 5 in^2 and 15 in^2, respectively. The Young's modulus was 30,450 ksi. In this work, a global buckling constraint was considered and the lower bound limit of the first buckling factor was λ_1^{Limit} =100. Figure 1 shows the three-dimensional (3D) view and top view of the un-deformed roof structure. SAP2000 Software (Computer and Structures 2011) was used to build the FE model and analyze the structure. Linear elastic buckling analyses were performed.

Two optimization methods were applied using two different metamodeling methods. Method 1 was the global RBF created using 51 sample points based on Latin Hypercube sampling. Method 2 was the adaptive RBF started with 21 initial sample points; one additional sample point was added in each subsequent iteration of the design optimization.

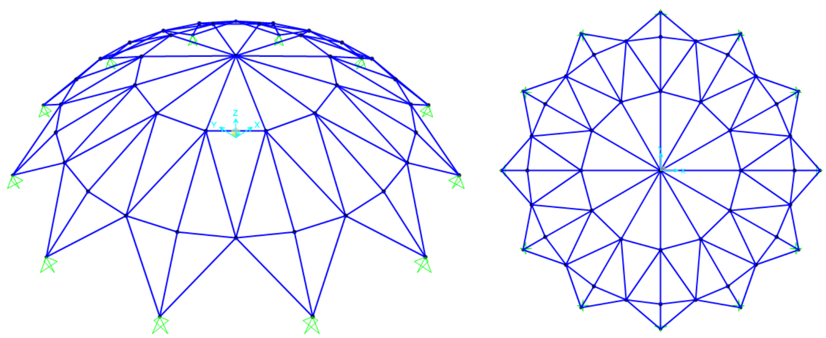

Figure 1. A 120-member roof dome structure.

Table 1. Optimal designs of the roof dome structure.

	RBF (51 samples)	Adaptive RBF (23 samples)	Lower bound	Upper bound
A_1 (in^2)	8.621	8.966		
A_2 (in^2)	5.000	5.000		
A_3 (in^2)	5.000	5.000	5.000	15.000
A_4 (in^2)	5.000	5.000		
A_5 (in^2)	5.000	5.000		
Volume (in^3)	137239	138388		
1st buckling factor (RBF)	100.000	100.000		
1st buckling factor (FE)	98.192	100.000		
1st buckling factor (% error)	1.8%	0.0%		

Table 1 shows the optimal designs of the roof structure using the two methods. For the adaptive RBF method, three iterations and a total of 21+1+1=23 sample points were used to achieve the final optimal design. The optimal design points obtained using the two optimization methods were verified using additional FE analyses, and the results were compared with those estimated using metamodels. Both methods worked well and the optimal designs were achieved.

5 SUMMARY AND CONCLUSIONS

An adaptive metamodeling method was developed and used in structural optimization. The adaptive metamodeling method combined an RBF metamodeling technique and a gradient-based numerical algorithm. The adaptive method was compared with a traditional global metamodeling approach in which the sample size was fixed. The new method improved the accuracy of the RBF metamodel during the optimization iterations, especially in the neighborhood of the optimal design point. To study the performance of the optimization method, a roof truss structure was optimized. The method worked well and accurate optimization results were obtained. The work provides an effective method for design optimization of complex engineering systems that require expensive FE analyses.

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