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A COMPARISON BETWEEN PROBABILISTIC AND POSSIBILISTIC NUMERICAL SCHEMES FOR STRUCTURAL UNCERTAINTY ANALYSIS

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Analysis of a structure is a crucial procedure to ensure its reliable design and performance. Because of complexities present in the closed-form structural analysis methods, numerical schemes have become the standard of practice. These schemes are generally performed deterministically. However, the input parameters defining the material and geometric properties may possess uncertainties. They can arise from various sources including modeling, manufacturing, and construction errors. The quantification of uncertainties is generally based on either probability theories (using random variables) or possibility theories (using interval and fuzzy variables). In this work, the finite-element-based probabilistic and possibilistic methods are discussed and compared. A case study of static and dynamic uncertainty analyses of a structure using the aforementioned schemes is performed. The results of those analyses suggest that the incorporation of uncertainty in the analysis provides a higher level of confidence. Moreover, they are compared for both accuracy and computational efficiency. Based on the results, it is observed that the determination of approach must be based on the problem complexity as well as the level of available information.

Keywords: Finite element method, Monte-Carlo simulations, Interval, Fuzzy.

1 INTRODUCTION

In reliable design of a structure, its stability and safety must be guaranteed. This may be achieved by predicting the overall behavior of the structure through analytical schemes over its design lifetime. These schemes construct structural models and consider the applied external static and dynamic loads to obtain the structures' response (deformations) and load effects (internal forces, moments, and stresses). Because of the limitations and complexities present in closed-form analytical methods, numerical and approximate methods such as finite element analysis are widely used (Hughes 1987).

Conventional structural analysis is performed deterministically. However, there exist uncertainties arising from numerous sources that may drastically alter the analytical results. Those uncertainties can stem from: a) physical imperfections (material and geometric properties), and b) modeling inaccuracies (approximate solutions, truncation errors, etc.)

Though conventional methods of structural design use a series of load amplification and strength reduction factors that are based on statistical models of historical data, the effects of uncertainties are not considered in structural analysis schemes used in practice. Therefore, for a reliable design, it is essential to incorporate uncertainties in analytical schemes.

In general, uncertainty is defined as the inability to predict the outcome of an event before it occurs. There exist two categories of uncertainties: 1) aleatoric, and 2) epistemic. Aleatoric uncertainties are inherent in the parameters and are irreducible. On the other hand, epistemic uncertainties stem from lack of knowledge and are reducible. The isomorphic paradigms of uncertainty analysis include probabilistic approaches based on stochasticity (using aleatoric random variables) as well as possibilistic set-based approaches (using epistemic interval and fuzzy variables). In terms of limitations, probabilistic approaches require a well-defined distribution of information, whereas possibilistic approaches require assumption on the bounds. In order to overcome those shortcomings, the polymorphic uncertainty analysis methods are introduced in recent years that combine both probabilistic and possibilistic approaches. The examples of those methods include imprecise probability, Dempster-Shafer functions, and fuzzy random variables.

In this work, the isomorphic-based approaches of structural uncertainty analysis using finite-element-based probabilistic and possibilistic methods are discussed and compared. For this comparison, three distinct methods of uncertainty analysis are utilized that includes: a) Stochastic finite element analysis based on Monte Carlo Simulations, b) Interval Analysis, and c) Fuzzy analysis. As a case study, a finite element model of a structural frame system is constructed and its static response and dynamic natural frequencies are determined. The problem initially assumes known values for material properties leading to obtaining deterministic results. Then, uncertainties in material properties are considered using random, interval, and fuzzy variables respectively. Following that, the corresponding uncertainty analyses for the aforementioned methods are performed. Finally, the results of each method are compared for their level of confidence and computational efficiency.

2 METHODOLOGY

2.1 Deterministic Finite Element Method

2.1.1 *Static problem*

The force equilibrium equation for a static problem is defined as a set of linear equations as in Eq. (1):

$$[K]\{U\}=\{F\} \tag{1}$$

where, $[K]$ is the stiffness matrix, $\{U\}$ is displacement vector (response), and $\{F\}$ is the load vector. The static displacements (response) are determined as in Eq. (2):

$$\{U\}=[K]^{-1}\{F\} \tag{2}$$

2.1.2 *Dynamic problem*

The equation of motion for an undamped, free vibration dynamic problem is defined as a set of linear homogeneous ordinary differential equations as in Eq. (3):

$$[M]\{\ddot{U}\}+[K]\{U\}=\{0\} \tag{3}$$

where, $[M]$ is the mass matrix, and $\{\ddot{U}\}$ is the acceleration vector.

The natural circular frequency is calculated by solving the following eigenvalue problem in Eq. (4):

$$([K]-\omega^2[M])\{\varphi\}=\{0\} \tag{4}$$

where, the values of ω are natural circular frequencies (eigenvalues) and the vectors $\{\varphi\}$ are the corresponding mode shapes (eigenvectors).

2.2 Structural Uncertainty Analysis

Analysis of a structure with uncertainty, using an enhanced numerical scheme such as uncertain finite element method, pertains to obtaining the structure's uncertain response and load effects (uncertain output) due to the applied uncertain forces/moments and displacements (uncertain input). Figure 1 shows the uncertain analysis scheme schematically.

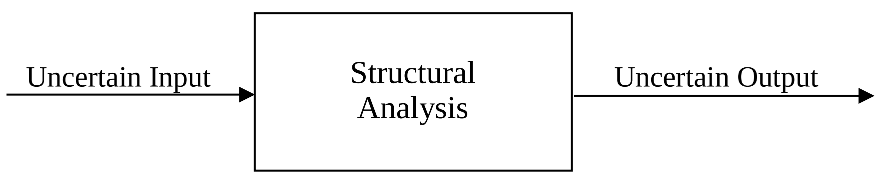

Figure 1. Structural uncertainty analysis scheme.

The following describes three methods of structural uncertainty analysis used in this work.

2.2.1 *Stochastic Finite Element Method (Monte Carlo Simulation)*

In Stochastic Finite Element Method, the input uncertainty is quantified by random variables. A random variable is defined by its prescribed probability function that yields the probability of existence of that variable in a given subset of the real space, Eq. (5). Figure 2 shows the probability density function of a random variable.

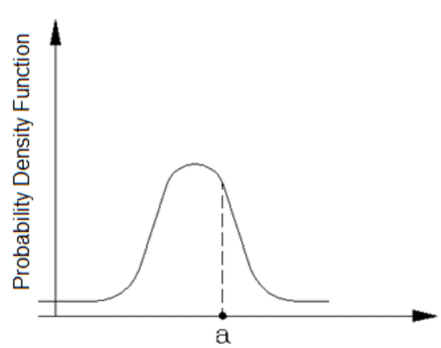

Figure 2. Random variable defined by its probability density function.

$$F_x(a)=P([x\le a])=\int_{-\infty}^{a} f(x)dx \tag{5}$$

where, F(x) is the cumulative probability function and f(x) is the probability density function in Eq. (5). In this approach, uncertainty analysis is performed by using Monte Carlo Simulations which are based on random sampling in an iterative process. Using the uncertain input as random variables in the analysis, the output values will possess intrinsic stochastic uncertainties. For structural design purposes, assuming bounded variables for the input, the quantities of interest are the upper and lower bounds of the uncertain output values.

2.2.2 *Interval Finite Element Method*

In Interval Finite Element Method, the input uncertainty is quantified by intervals (Moore and Lodwick 2003). A real interval is defined as a closed set bounded by extreme values, Eq. (6). Figure 3 shows a closed interval schematically.

$$\tilde{x} = [x_l, x_u] = \{x \in \Re \mid x_l \leq x \leq x_u\} \tag{6}$$

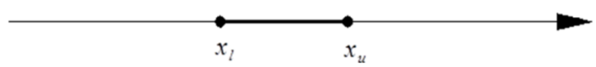

Figure 3. A closed interval.

Interval analysis has been used to quantify the uncertainty that arise from various sources including truncation and measurement errors. Using the uncertain input in the analysis as intervals, the output values will possess interval or set-based uncertainties. For structural design purposes, the quantities of interest are the upper and lower bounds of the uncertain output input values.

2.2.3 *Fuzzy Finite Element Method*

In Fuzzy Finite Element Method, the input uncertainty is quantified by fuzzy variables. A fuzzy variable is defined by its membership function, which yields the intervals of confidence for the corresponding levels of presumption (α-cuts) that are bound between 0 and 1 shown in Figure 4.

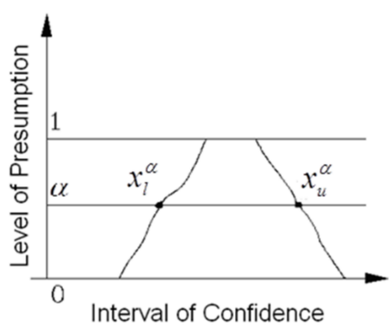

Figure 4. A Fuzzy membership function.

In analysis, defined by their membership functions the uncertain fuzzified input values are used to construct output values as fuzzy membership functions. For structural design purposes, the quantities of interest are the upper and lower bounds of the uncertain output input values for a desired level of presumption.

3 CASE STUDY

3.1 Problem Definition

As a case study, an uncertain 2D moment frame structure, with uncertainty in its material properties, is statically and dynamically analyzed using the aforementioned methods (Figure 5). It consists of three bays and 2 stories, all spaced at distance L. The beams and columns are W-shaped steel members (W18x40) with properties summarized in Table 1.

Table 1. Member properties.

Property	Value
A (in.^{2})	11.8
I (in.^{4})	612
ρ (lb/in.^{3})	0.2839
L (in.)	144

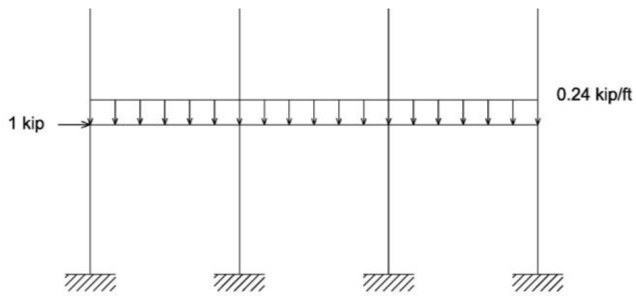

Figure 5. 2D frame with uncertain material properties.

3.2 Quantification of Uncertainty

The uncertainty is considered in the material properties of the structure in the modulus of elasticity. Independent uncertainties are considered for each member. In stochastic finite element method, bounded random variables ($\mu\pm\sigma$) are used, in which μ is the mean and σ is the standard deviation. In interval Finite element method, the lower and upper bounds for the modulus of elasticity for each member are considered. In Fuzzy finite element method, the membership function for modulus of elasticity for each member is quantified by six alpha cuts (Table 2).

Table 2. Quantification of material uncertainty.

Approach	Uncertain Variable	Uncertain Modulus of Elasticity (x 29,000) ksi
Bounded Random	$(\mu\pm\sigma)$	[0.90,1.10]
Interval	$[E_{Lower},E_{Upper}]$	[0.90,1.10]
	α=1	[1.00,1.00]
	α=0.8	[0.98,1.02]
Fuzzy	α=0.6	[0.96,1.04]
	α=0.4	[0.94,1.06]
	α=0.2	[0.92,1.08]
	α=0	[0.90,1.10]

3.3 Solution

Uncertain static and dynamic analyses on the frame structure are performed using stochastic, interval, and fuzzy finite element methods. Table 3 summarizes static results for lateral displacement of the roof at the right corner, as well as dynamic results for fundamental natural circular frequency (Modares *et al.* 2006, Vanmarcke and Grigoriu 1983).

Table 3. Summary of results.

Method		**Static Problem Horizontal Displacement at Point C**	**Dynamic problem Fundamental Natural Circular Frequency**
Stochastic Finite Element Method (Monte Carlo Simulation)		[0.0289,0.0334]	[4.8131,5.1791]
Interval Finite Element Method		[0.0282,0.0344]	[4.7455,5.2464]
Fuzzy Finite Element Method	α=1	[0.0310,0.0310]	[5.0022,5.0022]
	α=0.8	[0.0304,0.0316]	[4.9520,5.0520]
	α=0.6	[0.0298,0.0323]	[4.9012,5.1013]
	α=0.4	[0.0292,0.0330]	[4.8498,5.1501]
	α=0.2	[0.0287,0.0337]	[4.7980,5.1985]
	α=0	[0.0282,0.0344]	[4.7455,5.2464]

3.4 Observations

It is observed that the results of stochastic finite element method using Monte Carlo simulations are inner-bounds of interval finite element method. Also, interval finite element method, due to its set-based approach, is more computationally feasible than simulation processes. Moreover, fuzzy finite element method, because of discretization of information based on level of presumption, can yield more detailed information of the results.

Comparing probabilistic (stochastic) and possibilistic (interval and fuzzy) analyses shows that in presence of sufficient information, probabilistic methods can yield accurate results. However, in lack of sufficient information, possibilistic method can be used to obtain robust results.

4 CONCLUSIONS

In reliable design of a structure, the uncertainties must be considered in structural analysis schemes through probabilistic and possibilistic methods. Depending on the level of available information, probabilistic (with more information) or possibilistic method (with limited information) can be utilized. Thereby, the combination of probabilistic and possibilistic methods can be used for developing more reliable and robust analytical schemes.

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