



# ACCURATE MODELING OF CURVED STEEL I-GIRDER BRIDGE

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A curved and/or skewed steel I-girder bridge, in addition to the basic vertical shear and bending effects, will be subjected to torsional and warping effects. Thus, simplified hand calculation and line girder methods, might not be enough when bridges are to be analyzed. Refined methods, termed by AASHTO, have to be adopted. This paper has investigated the closeness and difference between curved bridge finite element models using 2-D gird and 3-D shell elements of I-girders, both are part of AASHTO refined method. Moreover, the results are calibrated by comparing analysis result with various two-dimensional and three-dimensional computations with varied curvature effects. It is concluded that when introducing torsional effects to finite element models, the modified torsional constant  $J$  with consideration of warping effect should be taken into the 2-D grid model as a refined model. When using 3-D shell elements as the refined model, stiffeners and connection plates play an important role of global model stiffness and should not be ignored, especially for sharp curved steel I-girder bridges.

*Keywords:* Torsional effect, Warping effect, Finite element analysis, Model optimization, Approximate method, Refined method, Stiffener, Connection plate.

## 1 INTRODUCTION

Steel bridges that designed and built in early times are primarily straight and simple-span, which can be analyzed and calculated by hand. Indeterminate structures, such as continuous span bridges, can now be easily handled by computer software. Those bridges, including simple span and continuous span bridges, are still mainly straight bridges that subject to major-axis shear and bending moment effects of the main girders. However, in addition to the vertical shear and bending effects, a curved and/or skewed girder bridge is subjected to torsional effects, which cause both normal stresses and shear stresses in steel girders. For I-shape girders, the torsions they carried are primarily by means of warping other than St. Venant torsion due to open cross-sectional geometry. Furthermore, the St. Venant torsional shear flow around the perimeter of the cross section can only develop relatively small force couples. The total normal stress in an I-shaped girder is a combination of any axial stress, major axis bending stress, lateral bending stress, and warping normal stress (Figure 1a). The total shear stress is the sum of vertical shear stress, horizontal shear stress, St. Venant torsional shear stress, and warping shear stress (Figure 1b). For non-skewed straight bridges, only the major axis bending stress and vertical shear stress are dominant; the other factors can be ignored in the design phase but have to be included in other load combinations for code checking.

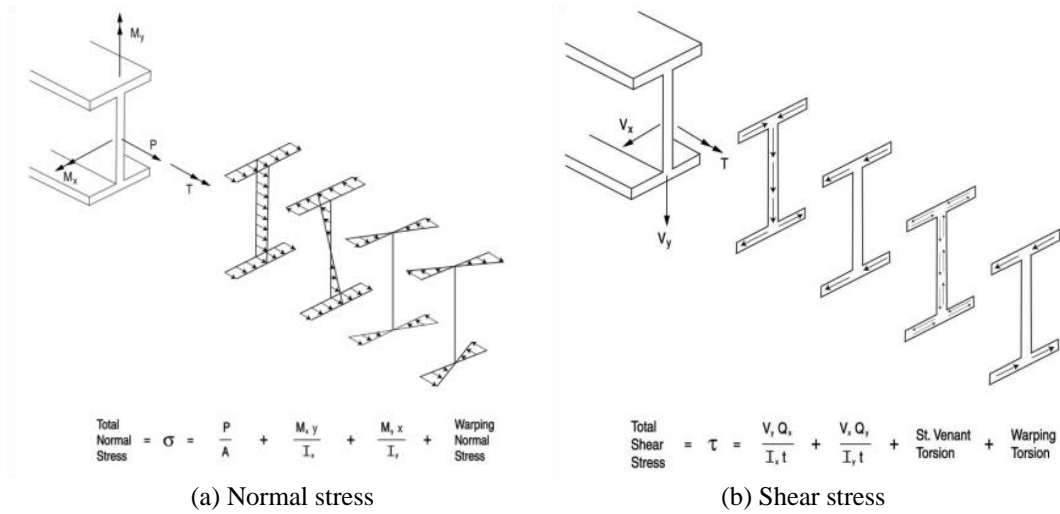


Figure 1. Illustration of the general I-girder normal and shear stresses which can occur in a curved or skewed I-shaped girder.

Generally, I-girders carry torsion through combination of pure torsion and restrained warping and the total torsional resistance can be expressed in Eq. (1). For the calculation of section properties, including  $C_w$ , refer to AISC Design Guide 9: Torsional Analysis of Structural Steel Members (AISC 2003).

$$T = GJ\theta' - EC_w\theta'' \quad (1)$$

Where:

$G$  = shear modulus of elasticity of steel;  $J$  = torsional constant of cross-section and can be approximated using Eq. (2) for rolled and built-up I shapes;  $E$  = modulus of elasticity of steel;  $C_w$  = warping constant of cross section and can be approximate as  $\frac{I_y h^2}{4}$  for rolled and built-up I shapes;  $I_y$  = lateral moment of inertia about Y-axis, and  $h$  = distance between centerlines of top and bottom flanges.

## 2 2D MODELING AND COMPUTATION

In two-dimensional grid bridge modeling, the distribution of forces through the system is highly dependent on member stiffness parameters such as  $EI_x$ ,  $EI_y$ ,  $GJ$ , and  $EC_w$ . The warping stiffness parameter  $EC_w$ , is not used in a generic structural analysis method base on the beam theory with six degrees of freedom per node. For special analyses, cross sectional warping deflection, the seventh degree of freedom can be included to consider the warping of thin-wall cross sections. Thus, the additional warping stiffness is required. For curved structure,  $EC_w$  is often the dominant contributor to the individual girder torsional stiffness. Without consideration of  $EC_w$ , the local twisting responses of the girders cannot be modeled accurately. On the other hand, a full 3D FEM analysis, in which the thin-wall sections are modeled by plane shell elements, bypasses the need for the modeling of warping stiffness within the single beam element used to model the girder in 2D grid analysis approaches (Fu and Wang 2014).

A rigorous solution of grid analysis to take care of the warping problem of a thin-wall beam requires an additional degree of freedom. Several researchers (e.g., Hsu *et al.* 1990, Fu and Hsu

1995) have included the warping deflection as the seventh degrees of freedom, in addition to the regular six DOFs, at each node for the curved beam analysis to consider the warping effect. For the case of partial warping restrained, an effective torsional constant,  $K_{eff}$ , was proposed by Fu and Hsu (1994) and later improved by Elhelbawey and Fu (1998) to consider warping effects in a regular six DOFs analysis. A simple, easy-to-apply effective torsional constant for the rotational stiffness of a restrained open section was developed to count for both the pure torsion and the warping torsion into account. This effective (equivalent) torsional constant,  $K_{te}$ , can be easily calculated and used for any generic finite-element structural analysis program.

The original torsional constant for most common structural shapes,  $J$ , can be approximated by Eq. (2)

$$J = \sum bt^3/3 \quad (2)$$

where  $b$  and  $t$  = width and thickness of the thin-wall elements, respectively. The effective (equivalent) torsional constant,  $K_{te}$ , developed by Fu and Hsu (1994), can be expressed as in Eq. (3)

$$K_{te} = J \cosh \frac{\lambda}{2} / \left\{ \cosh \frac{\lambda}{2} - 1.0 \right\} C \quad (3)$$

where  $\lambda^2 = GJ/EC_w$ , ( $\lambda = l/a$ , where  $a$  is used in AISC documents),  $C$  = correction factor that equals  $\{1.0/[1.0 + 2.95 (b/l)^2]\}$ ,  $l$  = unbraced length, and  $b$  = flange width.

Once the effective torsional constant is determined, the stiffness matrix for a grid structure can be derived by using the traditional straight beam method with three DOFs (torsional rotation, bending rotation and deflection) per node. The stiffness matrix of an element in grid model with warping partially restrained is as following in Eq. (4):

$$[K_e] = \begin{bmatrix} \frac{GK_{te}}{l} & 0 & 0 & -\frac{GK_{te}}{l} & 0 & 0 \\ 0 & \frac{4EI_y}{l} & \frac{-6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & \frac{6EI_y}{l^2} \\ 0 & \frac{-6EI_y}{l^2} & \frac{12EI_y}{l^3} & 0 & \frac{-6EI_y}{l^2} & \frac{-12EI_y}{l^3} \\ -\frac{GK_{te}}{l} & 0 & 0 & \frac{GK_{te}}{l} & 0 & 0 \\ 0 & \frac{2EI_y}{l} & \frac{-6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & \frac{6EI_y}{l^2} \\ 0 & \frac{6EI_y}{l^2} & \frac{-12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & \frac{12EI_y}{l^3} \end{bmatrix} \quad (4)$$

A similar study was done years later by the NCHRP Project 12-79 Report 725 (2012) with two equivalent equations with warping fixity at each end of a given unbraced length  $L_b$  (Eq. 5a) and warping fixity at one end and warping free boundary conditions (Eq. 5b) where  $J_{eq}$  is equivalent to  $K_{te}$  in Eq. (3).

$$J_{eq(fx-fx)} = J \left[ 1 - \frac{\sinh(pL_b)}{pL_b} + \frac{[\cosh(pL_b)-1]^2}{pL_b \sinh(pL_b)} \right]^{-1} \quad (5a)$$

$$J_{eq(s-fx)} = J \left[ 1 - \frac{\sinh(pL_b)}{pL_b \cosh(pL_b)} \right]^{-1} \quad (5b)$$

### 3 3D FINITE ELEMENT MODELING

The category 3D FEM analysis method is meant to encompass any analysis/design method that includes a computerized structural analysis model where the superstructure is modeled fully in three dimensions, including: modeling of girder flanges using line/beam elements or plate/shell/solid type elements; modeling of girder webs using plate/shell/solid type elements; modeling of cross frames or diaphragms using line/beam, link/truss, or plate/shell/solid type elements (as appropriate); and modeling of the deck using plate/shell/solid elements. This method is time-consuming and complicated, thus is arguably deemed most appropriate for use for complicated bridges (e.g., bridges with severe curvature, skew, or both, unusual framing plans, unusual support/substructure conditions, or other complicating features). 3D analysis methods are useful for performing refined local stress analysis of complex structural details (AASHTO/NSBA 2011).

When and how to use a refined 3D FEM analysis for engineering design is a controversial issue, and in the US such an approach has not been fully incorporated into the AASHTO specifications to date (AASHTO 2013). The typical AASHTO methodology for design is generally based on assessment of nominal (average) stresses calculated by simplified methods, such as  $P/A$  or  $Mc/I$ , and not localized peak stresses obtained by shell- or solid-based finite element models. Refined analysis can provide substantially more detailed and accurate information about the stress state of the structure. This could allow for more cost effective and reliable design but often comes with increased engineering effort and increased potential for error. However, if properly modeled, in the forensic or load test cases, such refined analysis is commonly adopted due to its refinement and accuracy. Researchers have done several modeling and analysis over the years. Hays Jr. *et al.* (1986) and Mabsout *et al.* (1997) modeled the deck slab using quadrilateral shell elements in plane with five degrees of freedom per node and the steel girders using 3D beam elements with six degrees of freedom per node (In-plane shell -beam model). Tarhini and Frederick (1992) *et. al.* used eight node linear solid brick elements with three displacement degrees of freedom in each node to model the concrete deck as a 3D Brick-shell model. Fu and Lu (2003) idealized bridge deck with isoperimetric quadrilateral shell elements and the reinforcement was modeled as a smeared two-dimensional membrane layer with isoperimetric plane stress element, and called 3D shell-shell model. In addition, 3D shell-beam model (Tabsh and Tabatabai 2001, Issa *et al.* 2000) and 3D brick-beam model (Ebeido and Kennedy 1996, Barr *et al.* 2001, Chen 1999, Sebastian and McConnel 2000) were used for three-dimensional finite element analysis of steel bridges (Fu and Wang 2014).

### 4 CASE STUDY

Bridge No. 27W18 is a two-span curved steel bridge with skewed angle and currently under design process. The general plan of the bridge is shown in Figure 2. Engineers and researchers had done structural analysis under dead load in the non-composite stage, i.e., the concrete deck is assumed with no stiffness but has weight on steel girders. In this study, seven analyses including four in two-dimensional and three in three-dimensional are computed and analyzed. For two dimensional calculations, engineers used two software, *DESCUS-I* and *STAAD*, based on various calculation methods, to compute reaction forces at bearings, moments and deflections for each girder in the bridge. *CsiBridge*, *ANSYS*, and *STAAD* are three software that used in 3D finite element analysis in this case and choose the 3D shell-shell model as the original model. It should be noted that in 3D finite element modeling for this case, modeling stiffeners and connection plates has significant influence on the results due to torsional effect by the curvature and skews.

The results of reaction forces and moments are listed and compared in Table 1, the deflection shapes of each girder are similar, and the maximum values are pretty close:

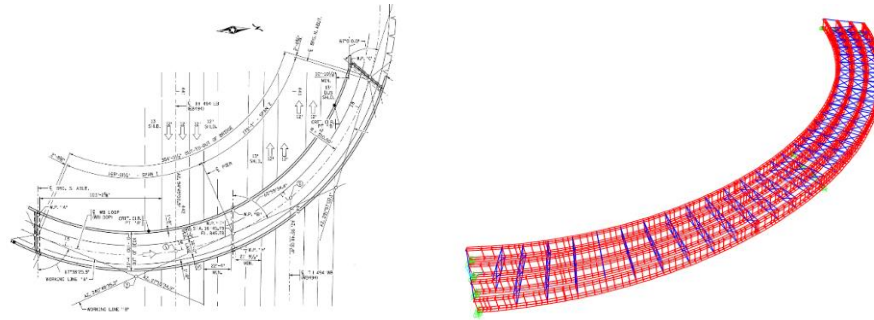


Figure 2. General plan and 3D FEM model of the bridge.

Table 1. Results of model analysis for bridge 27W18.

Girder No.	Analysis software and method	Moment (kip-ft)		Unfactored non-composite reaction (kips)		
		Maximum	Minimum	South Abutment	Pier	North Abutment
1	STAAD - NCHRP 725	5550.159	-9039.145	192.6	198.7	243.5
	STAAD - 2D grid	5678.432	-9113.4	200.5	184.2	243.2
	DESCUS-I	6086.24	-9639.28	202.9	207.3	247.1
	DESCUS-I - adjusted	5810.24	-9210.16	193.4	193	235.2
	STAAD - 3D	7719.243	-9549.975	187.5	198.1	223.9
	CsiBridge	6322.346	-9631.13	174.35	149.06	215.05
	ANSYS	5290.61	-9681.7	162.04	159.24	209.82
2	STAAD - NCHRP 725	4018.314	-9108.697	115.9	515.6	99.4
	STAAD - 2D grid	4048.466	-9248.765	101.1	525	100.3
	DESCUS-I	4252.88	-10077.28	113.5	534.6	113.9
	DESCUS-I - adjusted	4058.72	-9625.28	108.2	521.3	108.6
	STAAD - 3D	4697.415	-9928.611	118.7	492.8	128
	CsiBridge	4049.07	-9835.96	106.89	545.03	108.12
	ANSYS	4094.77	-9460.32	119.62	529.97	111.82
3	STAAD - NCHRP 725	2377.601	-7242.275	47.4	461.4	38.7
	STAAD - 2D grid	2311.716	-7320.26	51.1	472.1	33.4
	DESCUS-I	2324.08	-7635.52	53.7	479.8	35.4
	DESCUS-I - adjusted	2216.24	-7289.84	51.3	456.1	34
	STAAD - 3D	1919.312	-8300.873	49.8	471.4	34.2
	CsiBridge	1904.45	-7433.59	56.35	438.11	34.15
	ANSYS	1889.64	-7983.89	52.46	434.52	31.22
4	STAAD - NCHRP 725	1125.841	-5709.293	47.2	370.7	43.2
	STAAD - 2D grid	1113.356	-5651.61	49.2	367.5	46.4
	DESCUS-I	1059.44	-5828.4	51	402.6	46.9
	DESCUS-I - adjusted	1008.32	-5559.52	48.6	384.5	44.7
	STAAD - 3D	642.579	-6590.325	45.3	410.7	35.3
	CsiBridge	748.93	-5460.8	50.96	406.06	49.67
	ANSYS	898.74	-6723.443	46.8	407.22	47.41

## 5 CONCLUSIONS

The curved skew I-girder bridges have different behavior under loading compared with straight bridges. This difference would be caused most due to torsional effects on the open section of I girder. Thus, for accurate modeling, the effective torsional constant should be taken into consideration for 2D analysis, and the connection plates and stiffeners should be modeled and added to the steel girders when doing 3D finite element analysis for curved and/or skewed bridge.

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