Interdependence between Structural Engineering and Construction Management
*Edited by* Ozevin, D., Ataei, H., Modares, M., Gurgun, A., Yazdani, S., and Singh, A.

ISBN: 978-0-9960437-6-2

# STRESS INTENSITY FACTOR ANALYSIS OF AN EDGE CRACK PERPENDICULAR TO THE INTERFACE IN BI-MATERIALS

YONGKANG HOU, SHUO ZHANG, SHUJIN DUAN, and RUIMEI AN

*Research Institute of Structural Engineering, Shijiazhuang Tiedao University, Shijiazhuang, China*

The purpose of this paper is to investigate the stress intensity factor in the problem of a crack perpendicular to the interface in bi-materials. Based on the theory of an edge crack perpendicular to the interface between two dissimilar isotropic half-planes, the stress intensity factor was yield out from the stress singularity eigen-equation. The stress intensity factor for a composite beam segment with an edge crack under bending and tension was computed by the ordinary finite element and the singular finite element, and it was found that the singular element method is more accurate and applicable than the ordinary element method. The influence of the distance from the crack tip to the interface and the material's mechanical properties to the stress intensity factor were analyzed. The results show that the stress intensity factor increases first, and then decreases with decreasing of the distance from the crack tip to the interface, and the stress intensity factor increases with increasing of the elastic modulus of the cracked material.

*Keywords*: Fracture mechanics, Ordinary element method, Singular element method.

## 1 INTRODUCTION

Composite materials have been increasingly used in practical engineering applications, the fracture and failure of composite materials have attracted much attention of investigators (Xu 2006), but the most researches are devoted to the crack that along the interface or the tip at the interface (Williams 1959, Duan *et al.* 1989, Xu 2006). The problem of crack perpendicular to the interface was researched by Zak (1963) firstly, he used the eigenfunction expansion method to analyze the stress singularity ahead of a crack tip, which is perpendicular to and terminating at the interface. Cook and Erdogan (1972) used the Mellin transform method to derive the governing equation of a finite crack perpendicular to the interface and obtained the stress intensity factors. Qin and Noda (2002) used supersingular integral equation to analyze the problem of 3D crack perpendicular to the interface and obtained the numerical solution of stress intensity factor. Bouhala *et al.* (2013) studied the crack with the tip at the interface by the finite element method and computed the stress intensity factor by the physical method.

All the above studies are almost about the calculation method of stress intensity factor of crack perpendicular to the interface, the influence factor to the stress intensity factor needs to be studied further. In this paper, the stress intensity factor is yield out by the by the ordinary finite element and the singular finite element, and the influence of the distance from the crack tip to the interface and the material's mechanical properties to the stress intensity factor are analyzed.

## 2 THE STRESS INTENSITY FACTOR OF CRACK PERPENDICULAR TO THE INTERFACE IN BIMATERIALS

The model of crack perpendicular to the interface in bi-materials is established, as shown in Figure 1 (here, $E$ is elastic modulus, $v$ is Poisson's ratio).

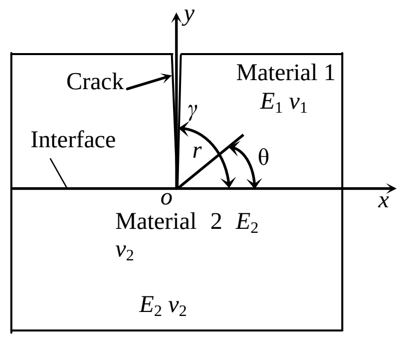



Figure 1. Schematic of crack perpendicular to the interface in bi-materials.

Following Zak and Williams (1963), we assume a stress function $\Phi(r, \theta)$, which satisfies the biharmonic equation, Eq. (1):

$$\nabla^2\nabla^2\Phi(r,\theta)=0 \tag{1}$$

The function of materials 1 and materials 2 take the form as shown in Eq. (2) and Eq. (3):

$$\Phi_1(r,\theta)=r^{\lambda+1}\left[A_1\cos(\lambda+1)\theta\text{-}B_1\sin(\lambda+1)\theta+C_1\cos(\lambda-1)\theta\text{-}D_1\sin(\lambda-1)\theta\right] \tag{2}$$

$$\Phi_2(r,\theta)=r^{\lambda+1}\left[A_2\cos(\lambda+1)\theta-B_2\sin(\lambda+1)\theta+C_2\cos(\lambda+1)\theta-D_2\sin(\lambda-1)\theta\right] \tag{3}$$

The stress and displacement components can be obtained from Eq. (2) and Eq. (3). By satisfying the continuity conditions at the interface and the stress-free boundary conditions at the crack surfaces, the obtained homogeneous equations about $\lambda$ should be zero to determining $\lambda$:

$$\begin{aligned}&\left[1-\beta+2\alpha\left(1+\alpha-\alpha\lambda^2+\beta\left(\lambda^2-1\right)\right)\right]\sin\pi\lambda-\alpha\left(\beta-\alpha\right)\lambda_2\cos\left(\pi\lambda-2\gamma\right)\\&-\left(\alpha+\alpha^2-\alpha\beta\right)\sin\left(\lambda\pi-2\lambda\gamma\right)-\beta\lambda\sin2\gamma-\left(1+\alpha\right)\left(\alpha-\beta\right)\sin\left(2\lambda\gamma+\lambda\pi\right)\\&-\left(\alpha\beta\lambda^2-\alpha2\lambda^2\right)\sin\left(\pi\lambda+2\gamma\right)=0\end{aligned} \tag{4}$$

When the crack perpendicular to the interface, $\gamma = \pi/2$, the Eq. (4) can be written as:

$$\sin\lambda\pi\left[\lambda^2\left(-4\alpha^2+4\alpha\beta\right)+2\alpha^2-2\alpha\beta+2\alpha-\beta+1+\left(2\alpha^2-2\alpha\beta+2\alpha-2\beta\right)\cos\lambda\pi\right]=0 \tag{5}$$

Where $\alpha = (\mu_1/\mu_2\text{-}1)/(1+\kappa_1)$, $\beta = (\mu_1/\mu_2)(1+\kappa_2)/(1+\kappa_1)$, $\mu$ is shear modulus, $\kappa = 3\text{-}4v$ for plane strain and $\kappa = (3\text{-}v)/(1+v)$ for plane stress, $v$ is Poisson's ratio.

Refer to the definition of stress intensity factor in material stress singularity, using the stress of crack perpendicular to the interface in bi-materials, the $K_I$ can be written as:

$$K_{\mathrm{I}} = \lim_{r \to 0} \sqrt{2\pi} r^{1-\lambda} \sigma_{\theta}(r, \theta) \quad (\theta = -\frac{\pi}{2}) \tag{6}$$

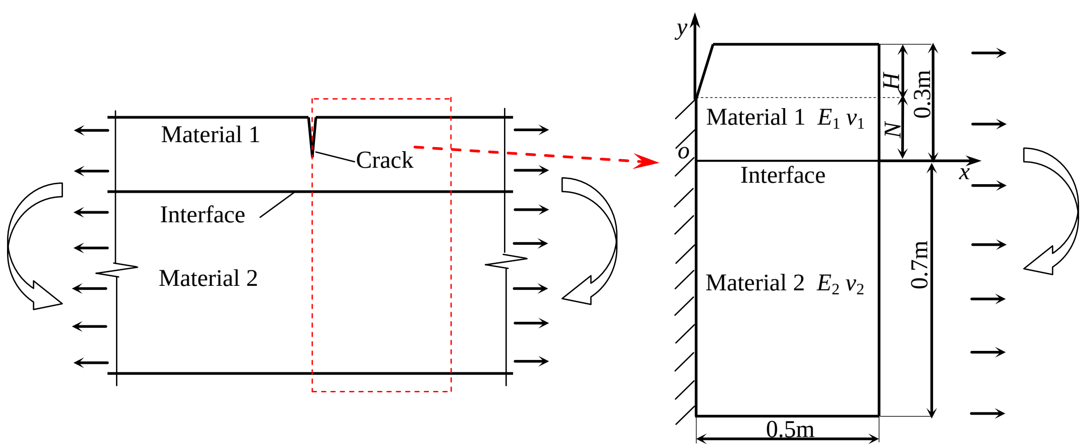
Substituting the expression of stress into Eq. (6), the expression of stress intensity factor is as follow in Eq. (7):

$$K_{\mathrm{I}} = \lim_{r \to 0} \sqrt{2\pi} r^{\lambda-1} \lambda(\lambda+1) F_{\lambda} \quad (\theta = -\frac{\pi}{2}) \tag{7}$$

## 3 THE SIMULATION MODEL

A partial beam segment of 1 m long with a central edge-crack is cut from a beam structure under tension and negative bending for analysis, and the half plane is selected for modeling, as shown in Figure 2.

Where the width of material 1 and material 2 are 0.3 m and 0.7 m respectively, and the length is 0.5 m. $E_1$, $E_2$ are elastic modulus, $n_e$ is the elastic modulus ratio ($n_e = E_1/E_2$ and $n_e \leqslant 1$), the material 1 is weaker; $v_1$, $v_2$ are Poisson's ratio, $\eta$ is the ratio of Poisson's ratio ($\eta = v_1/v_2$ and $\eta \leqslant 1$). The crack lies in material 1 and expands to the interface. $N$ is the distance from the crack tip to the interface, and $H$ is the distance from the crack tip to the top surface ($H + N = 0.3$ m).




(a) The beam under bending and tension

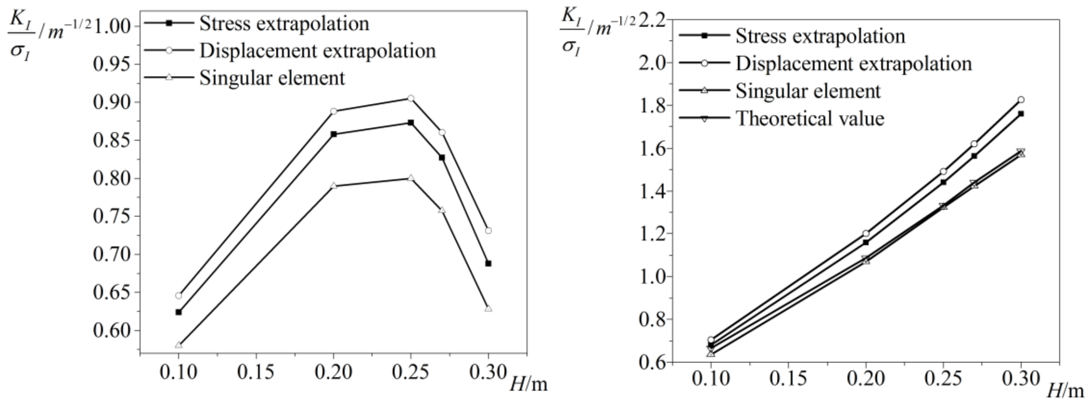
(b) The modeling

Figure 2. The problem under consideration.

The above problem of a crack perpendicular to the interface in bi-materials under bending or tension is examined respectively, and the stress intensity factors are obtained by using the ordinary finite element and the singular finite element.

### 3.1 Stress Intensity Factor Analysis under Tension

In the present study, the model under tension is analyzed, $H$ is taken as 0.10 m, 0.20 m, 0.25 m, 0.27 m and 0.3 m respectively, and the $K_{\mathrm{I}}/\sigma_{\mathrm{I}}$ is calculated, one can obtain the tendency of $K_{\mathrm{I}}/\sigma_{\mathrm{I}}$ to change with $H$, as shown in Figure 3.

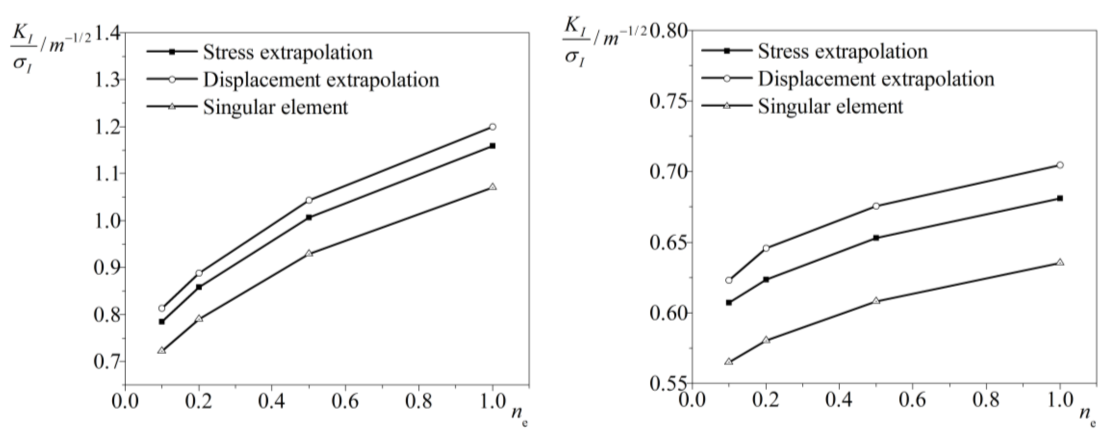



(a) $n_e$ = 1.0 $\eta$ = 1.0 (b) $n_e$ = 0.2 $\eta$ = 1.0

Figure 3. The stress intensity factor versus the *M* under tension.

From Figure 3, it is shown that the error of the calculation result between the finite element method and the theory method is smaller, in which the singular element method is more accurate than the ordinary element method; when $n_e$ =1.0, or $n_e$ =0.2, but the crack tip is far away from interface, the model can be seen as homogenous material, the stress intensity factor increases with increasing of the *H*; when $n_e$ = 0.2, and the crack lies in weaker material, if the crack tip is closer to the interface, due to the influence of the stiffer material, the stress intensity factor decrease with increasing of the *H*.

Continue to analyze the model, and the elastic modulus ratio is taken as 0.1, 0.2, 0.5, and 1.0 respectively, calculated the value of $K_I/\sigma_I$, to obtain the tendency of $K_I/\sigma_I$ to change with $n_e$, as shown in Figure 4.




(a) $N$ = 0.1 $\eta$ = 1.0 (b) $N$ = 0.2 $\eta$ = 1.0

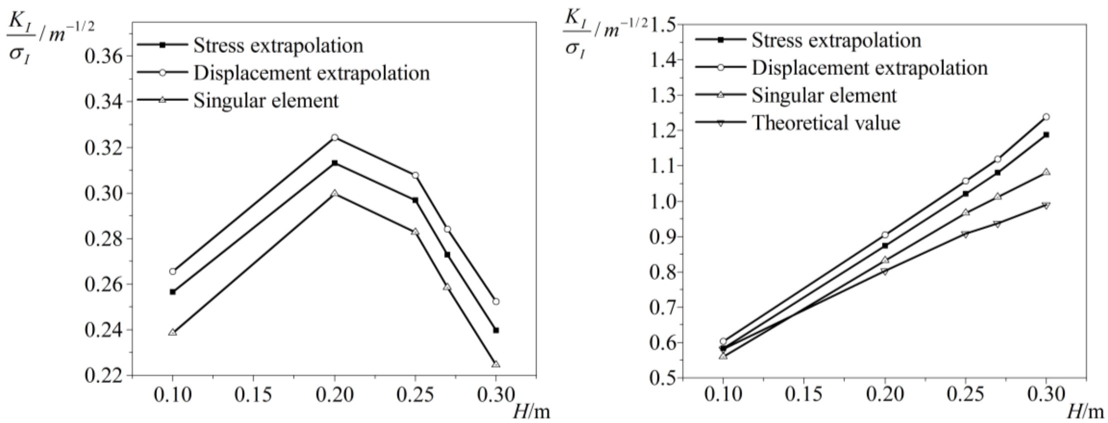
Figure 4. The stress intensity factor versus the $n_e$ under tension.

From Figure 4, it is shown that the stress intensity factor increases with increasing of the elastic modulus of the cracked material, and the stress intensity factor decrease with the increasing of the difference of the elastic modulus between constituent materials.

### 3.2 Stress Intensity Factor Analysis under Bending

The stress intensity factor of crack perpendicular to the interface in bi-materials under bending, is analyzed, the parameters are the same as the tension.

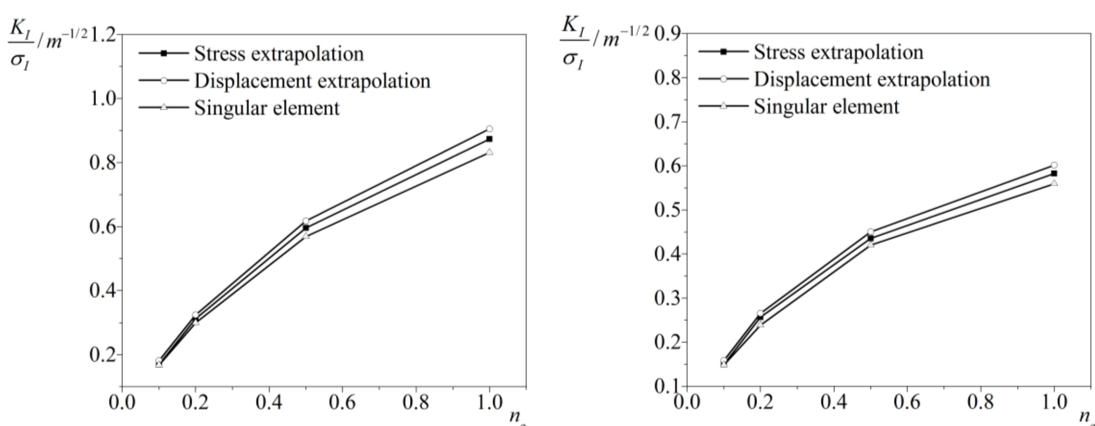
Figure 5 shows the tendency of $K_I/\sigma_I$ to change with $H$. From Figure 5, it is shown that the error of the calculation result between the finite element method and the theory method are smaller, in which the singular element method is more accurate than the ordinary element method. The tendency of $K_I/\sigma_I$ to change with $H$ is the same as the tension.




(a) $n_e = 1.0$ $\eta = 1.0$ (b) $n_e = 0.5$ $\eta = 1.0$

Figure 5. The stress intensity factor versus the $M$ under bending.

Figure 6 plots the tendency of $K_I/\sigma_I$ to change with $n_e$. From Figure 6, one observes that the stress intensity factor increases with increasing of the elastic modulus of the cracked material, and the law of change is the same as the tension.




(a) $N = 0.1$ $\eta = 1.0$ (b) $N = 0.2$ $\eta = 1.0$

Figure 6. The stress intensity factor versus $n_e$ under bending.

## 4 CONCLUSIONS

In the paper, the stress intensity factor of crack perpendicular to the interface in bi-materials was yielded out theoretically, and the stress intensity factor was analyzed by the ordinary finite element and the singular finite element, and from the numerical results some conclusions can be drawn as follows:

(i) Compared with ordinary element method, the result of the singular element method is more accurate and close to theoretical value;

(ii) When the edge crack lies in a weaker material, if the crack tip is far away from interface, the model can be considered as homogenous material, the stress intensity factor increases with decreasing of distance from the crack tip to the interface; if the crack tip is closer to the interface, the stress intensity factor decreases with decreasing of distance from the crack tip to the interface due to the effect by the stiffer material.

**Acknowledgments**

The authors greatly acknowledge the work support from the Hebei Provincial Natural Science Foundation, P. R. China (No. A2015210029) and the Hebei Provincial Graduate Innovation Fund Project, P. R. China (No CXZZBS2017132).

## References

Bouhala L., Shao Q., Koutsawa Y., Younes, A., Nunez, P., Makradi, A., and Belouettar, S., An XFEM Crack Tip Enrichment for a Crack Terminating at a Bi-Material Interface, *Engineering Fracture Mechanics*, 102(2), 5164, 2013.

Cook, T. S., and Erdogan, F., Stresses in bonded materials with a crack perpendicular to the interface. *International Journal of Engineering Science*, 10(8), 677697, 1972.

Duan, S., Yazaki, H., and Nakagawa, K., A Crack at the Interface of an Elastic Half Plane and a Rigid Body, *Engineering Fracture Mechanics*, 32, 573-580, 1989.

Qin, T. Y., and Noda, N. A., Analysis of a Three-Dimensional Crack Terminating at an Interface Using a Hyper-Singular Integral Equation Method, *ASME Journal of Applied Mechanics*, 69(5), 626631, 2002.

Williams, M. L., The Stresses Around a Fault of Cracks in Dissimilar Media, *Bulletin of The Seismological Society of America*, 49, 199-204, 1959.

Xu, J., *The Mechanics of Interface*, Beijing: Science Press, 17, 2006.

Zak, A. R., and Williams, M. L., Crack Point Stress Singularities at a Bi-Material Interface, *Journal of Applied Mechanics*, 30(1), 142-143, 1963.