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A STRUT AND TIE MODEL FOR FRCM-STRENGTHENED REINFORCED CONCRETE DEEP BEAMS

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Exiting literature revealed that fabric reinforced cementitious matrix (FRCM) is a promising material for the strengthening of shear deficient reinforced concrete (RC) beams. However, most of the available experimental studies are devoted to the use of FRCM system for the strengthening of slender beams and limited literature is available on the strengthening of deep beams using FRCM system. Moreover, there is no available literature on the analytical modelling of FRCM-strengthened deep beams. In this paper, a simple strut and tie model (STM) has been used to predict the ultimate load carrying capacity of RC deep beams strengthened in shear using FRCM system. The model accounts for the internal transverse reinforcement ratio and axial rigidity of the FRCM system. The proposed model is validated against an experimental results of RC deep beams strengthened with different types of FRCM system available in the literature.

Keywords: Fabric reinforced cementitious matrix, Strengthening, Shear capacity.

1 INTRODUCTION

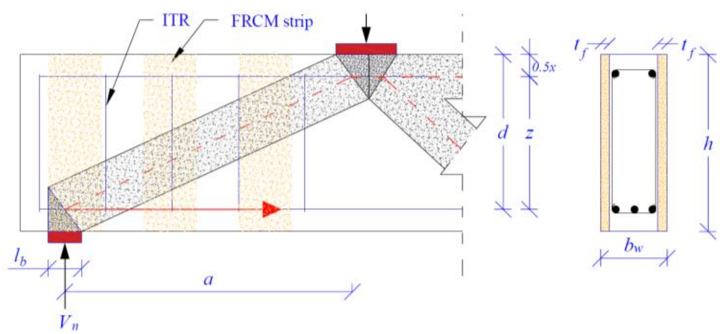
One of the major problems faced in construction industries worldwide is an early deterioration of reinforced concrete (RC) structures encountering their life cycle (Younis *et al.* 2018).

Fiber reinforced polymer (FRP) system has been widely used for the strengthening of deteriorated structures owing to their favorable properties over the traditional strengthening system (Aly *et al.* 2006, Ebead 2015, Ebead and Saeed 2013, 2014). However, FRP possess some drawbacks, such as incompatibility with the concrete substrate and failure at high temperature due to reliance on epoxy adhesives. To overcome these problems, fabric reinforced cementitious matrix (FRCM) has been introduced by replacing the epoxy adhesives with cementitious mortar and fibers with the textile fabrics. Exiting literature revealed that FRCM can be used to significantly enhance the flexural and shear capacity of RC beams (Ebead *et al.* 2017, Ebead and Wakjira 2018a, 2018b, Elghazy *et al.* 2017, 2018, Pino *et al.* 2017, Tetta *et al.* 2016, Wakjira and Ebead 2018c, 2018a, Younis *et al.* 2017) possessing a good bond with the concrete (Younis and Ebead 2018). However, most of the available experimental studies are devoted to the use of FRCM system for the strengthening of slender beams and the literature available on the FRCM-strengthened deep beams are scarce (Azam *et al.* 2018, Tetta *et al.* 2018, Wakjira and Ebead 2018b, 2019). Moreover, there is no available literature on the analytical modelling of FRCM-strengthened deep beams. ACI-ASCE Committee 445 (Joint ACI-ASCE Committee 445 1998) defined deep beams as beams in which the ratio of the critical shear span to effective depth (*a/d*) is less than 2.50. In this type of beams, unlike slender beams in which the combined shear and bending action transfers the load to the support, the load is transferred to the support through the compressive stresses by arch action (He *et al.* 2012). A strut-and-tie model (STM) has shown to be an effective design method for this type of beam.

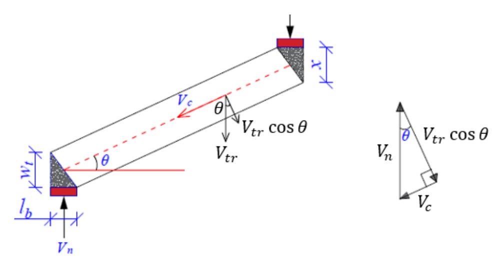
In this paper, a strut and tie model is used to predict the ultimate load carrying capacity of FRCM-strengthened RC deep beams. The model is then validated against an experimental data of FRCM-strengthened deep beams available in the literature.

2 STM FOR FRCM-STRENGTHENED RC DEEP BEAMS

The failure in deep beams is approximated to follow the shortest possible path (Tuchscherer *et al.* 2016). Figures 1a and 1b show the proposed STM for FRCM-strengthened beams in which the load is assumed to be transferred directly to the support in a form of a compressive strut. The diagonal compressive strut may fail because of concrete splitting caused by the diagonal tension stress or the crushing of concrete caused by formation of diagonal compressive stress. The latter is resisted by concrete compressive strength of the diagonal strut, while the diagonal tension stress is mainly resisted by transverse reinforcement; i.e., internal transverse reinforcement (ITR) and external transverse reinforcement (FRCM) as shown in Figure 1b.



(a)



(b)

Figure 1. Detail of STM for FRCM-strengthened beams within the critical shear span: (a) layout of STM and (b) load transfer and equilibrium of forces.

From the equilibrium of forces on the strut, Figure 1b, the nominal shear capacity (V_n) of FRCM-strengthened beam is given in Eq. (1).

$$V_n = V_c \sin\theta + V_{tr}\cos^2\theta \qquad (1)$$

where, V_c and V_{tr} are the shear force in the concrete compressive strut and the shear strength provided by the transverse reinforcement, which are ITR and FRCM reinforcement. The inclination (θ) of the compressive strut from the horizontal can be determined as follows.

$$\theta = \tan^{-1}(z/a) \tag{2}$$

Where

a is the critical shear span;

z is the lever arm determined from the cracked rectangular section of singly reinforced RC beam as given in Eq. (3).

$$z = d - \frac{kd}{3} \tag{3}$$

Where

d is the effective depth of the section;

kd is the position of the neutral axis from the extreme compression side of the beam.

The factor *k* is determined from the moment-curvature relationship of singly reinforced RC beams as given in Eq. (4) (Park and Kuchma 2007).

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{4}$$

Where ρ is the flexural reinforcement ratio ($A_s/b_w d$) and *n* is the steel-concrete modular ratio, which is the modulus of elasticity of steel (E_s=200,000 MPa) to concrete (E_c). The modulus of elasticity of concrete is given by Eq. (5) according to (ACI Committee 318 2014).

$$E_c = 4700\sqrt{f_c'} \tag{5}$$

The force in the concrete compressive strut (V_c) is given by Eq. (6).

$$V_c = f_{ce} A_{st} \tag{6}$$

Where,

f_{ce} is the effective concrete strength;

A_{st} is the smaller of cross-sectional area of the strut at the two ends.

According to (ACI Committee 318 2014) the effective concrete strength is the smaller of the effective compressive strength of the concrete in strut ($f_{ce,st}$) and nodal zone ($f_{ce,n}$) as given in Eq. (7) and Eq. (8), respectively.

$$f_{ce,st} = 0.85\beta_s\, f_c' \tag{7}$$

Where, β_s is the factor that depends on the shape of the strut, which is unity for prismatic strut considered in this study.

$$f_{ce,n} = 0.85\beta_n\, f_c' \tag{8}$$

Where, β_n is given by Eq. (9) below as per (ACI Committee 318 2014).

$$\beta_n = \begin{cases} 1.0, & \text{nodal zone bounded by strut only} \\ 0.80, & \text{nodal zone anchoring one tie} \\ 0.60, & \text{nodal zone anchoring more than one tie} \end{cases} \tag{9}$$

Thus, β_n is 0.80 for the beams considered in this study. Area of the strut is given by Eq. (10).

$$A_{st}=b_w a_{st} \tag{10}$$

Where, a_{st} is the minimum of the width of the strut at the two ends given by Eq. (11).

$$a_{st}=\min\left(l_b \sin\theta + w_t \cos\theta,\ l_p \sin\theta + x \cos\theta\right) \tag{11}$$

Where,

l_b is the with of the support plate;

l_p is the width of the loading plate;

w_t is the depth of the tie, which is taken as twice the distance between the centroid of flexural reinforcement and extreme tension side of the beam;

x is the depth of the top horizontal strut, which is taken as twice the distance between the centroid of the compression zone in the moment-curvature and extreme compression side of the beam (2kd/3). The shear strength provided by the transverse reinforcements (V_{tr}) is the sum of the shear provided by the ITR (V_{itr}) and FRCM reinforcement (V_f) as given in Eq. (12).

$$V_{tr}=f_{itr}A_{itr}+f_f A_f \tag{12}$$

Where,

f_{itr} and A_{itr} are the tensile stress and area of ITR;

f_f and A_f are the effective tensile stress and area of FRCM reinforcement.

The effective tensile stress of the FRCM system is determined as the product of its elastic modulus (E_f) and effective strain (ε_f) as given in Eq. (13).

$$f_f=E_f\varepsilon_f \tag{13}$$

The effective strain in FRCM system is limited to the minimum of 0.004 and 75% of its ultimate strain as per (ACI Committee 549 2013). Therefore, the shear capacity of FRCM-strengthened beam is given by Eq. (14).

$$V_n=f_{ce}A_{st}\sin\theta+f_{itr}A_{itr}\cos^2\theta+E_f\varepsilon_f A_f\cos^2\theta \tag{14}$$

Finally, the ultimate load carrying capacity of the beam can be determined based on the loading condition.

3 VERIFICATION OF THE MODEL

The proposed model is validated against an experimental database of RC deep beams strengthened with FRCM system available in the literature (Azam *et al.* 2018, Tetta *et al.* 2018, Wakjira and Ebead 2018b). Azam *et al.* (2018) tested rectangular RC beams of dimensions 1700 × 250 × 400 mm (length × width × height) with critical shear span to effective depth (*a/d*) ratio of 1.62. The beams were strengthened with externally bonded cement-based composites and internally reinforced with or without internal ITR. The beams tested by (Wakjira and Ebead 2018b) were of 2100 mm long rectangular beams with cross-sectional dimensions 150 × 330 mm (width × height) tested under three-point bending with *a/d* ratio of 1.96. The average 28-day compressive strength of concrete used to cast the beams was 30 MPa. The beams were strengthened with both continuous and discontinuous configurations of three different types of FRCM composites (carbon, glass, PBO) applied using a near surface embedded technique. The beams tested by (Tetta *et al.* 2018) had rectangular cross section of dimensions 1677 × 102 × 203 mm) (length × width × height) tested under three-point loading with *a/d* ratio of 1.6. The beams were strengthened with an externally bonded carbon FRCM. However, the slender beams with *a/d* ratio greater than 2.50 tested by (Tetta *et al.* 2018) are not included in this study.

The ratio of the theoretically predicted to experimental load carrying capacity (P_{th}/P_{ex}) is depicted in Figure 2. This ratio ranged between 0.81 and 1.11 with an average of 0.94 and standard deviation of 0.09 as shown in Figure 2. The correlation of determination (R^2) between the experimental and theoretically predicted ultimate load carrying capacity was 0.96. This observation shows that the proposed STM can predict the load carrying capacity of the FRCM-strengthened deep beams with an acceptable accuracy.

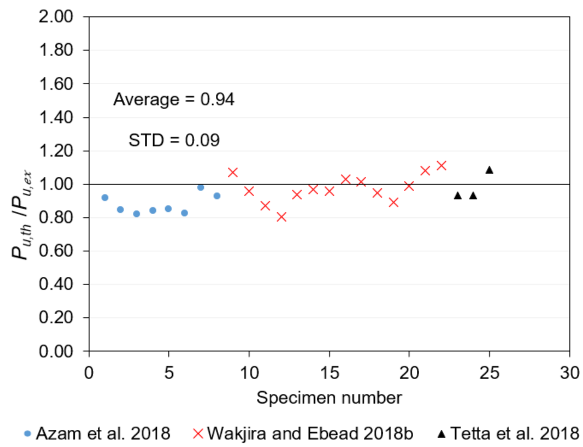


Figure 2. Verification of the model against experimental data.

4 CONCLUSIONS

A simple strut and tie model was used to predict the ultimate load carrying capacity of FRCM-strengthened RC deep beams. The model accounts for internal transverse reinforcement ratio and axial rigidity of the FRCM system. An experimental database of FRCM-strengthened deep beams available in the literature was used to validate the model and a reasonable agreement between the experimental and theoretically predicted ultimate load of the beams was obtained. The correlation of determination (R^2) between the experimental (P_{ex}) and theoretically predicted (P_{th}) ultimate load carrying capacity was 96%, while the average of P_{th}/P_{ex} ratio was 0.94 with a standard deviation of 0.09.

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