AN RBF BASED HIGH-DIMENSIONAL MODEL REDUCTION APPROACH IN BUILDING OPTIMIZATION

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Building optimization has gained importance with the recent push to create the most economical and efficient buildings possible. As the effects of optimization are a function of the building size, it is crucial to understand and further develop optimization techniques for large-scale building structures. Practical structural optimization of buildings requires the use of a structural analysis software package and an iterative optimization procedure. As a result, finite element (FE) software shall be linked with an optimization solver. It is an expensive process which requires extensive computer coding. Alternative methods are available, including metamodeling methods, which are used to create simple and approximate functions based on complex FE simulations. In this study, the approximate functions are generated using a high-dimensional model representation (HDMR) framework. The HDMR framework is a model reduction approach and is found to be very accurate for different functions. The component functions of HDMR are expressed using augmented radial basis functions (RBFs). To further improve the numerical efficiency of the metamodels and reduce the total required number of structural analyses, a few different HDMR sampling approaches are investigated, including one static approach and two iterative strategies. An existing nonlinear programming (NLP) solver is employed in the design process. To illustrate the proposed approach, a three-dimensional building structure is selected as a numerical example. The numerical optimization is conducted to reduce the torsional response of the building. The proposed optimization method works very well and the results from different HDMR techniques are compared.

Keywords: Augmented radial basis function, Sampling approach, Nonlinear programming (NLP), Finite element (FE).

1 INTRODUCTION

Design optimization of practical building structures has attracted considerable attention in recent years (Zou et al. 2007, Zou et al. 2018). In order to achieve an optimal design, an iterative numerical algorithm is applied and repeated structural analyses are required (Arora and Wang 2005). It can be computationally expensive, if an existing FE software is called many times to analyze structural responses in an optimization loop. An alternate approach has been developed to explore approximate models, i.e., metamodels, in structural optimization. To use metamodels in a structural optimization problem, the actual structural response functions are approximated. In the literature, many different metamodeling techniques have been developed and investigated (Bi et al. 2010, Simpson et al. 2011, Wang and Fang 2018, Wang et al. 2020). Among all the available methods, HDMR is an efficient framework to decompose a function into component
functions (Tunga and Demiralp 2005, Wu et al. 2019). Standard first-order HDMR methods are very efficient, but are not always accurate. The high-order HDMR methods usually require a lot more sample points, which is not preferred for large-scale problems involving time-consuming FE simulations. There is a need to develop and study more efficient HDMR methods, that can enhance the model accuracy of first-order HDMR methods, without significantly increase the computational cost, i.e., sample size.

This work focuses on the application of a new metamodeling methods to structural design optimization of large-scale buildings. An alternative HDMR method is studied, and the selection of samples is based on static and dynamic sampling approaches. For the iterative methods, once the initial RBF-HDMR metamodel is created, new sample points can be located based on optimal design of the previous iteration. As the engine of the optimization approach, a conventional gradient-based NLP solver is used in each design iteration. The proposed approach is applied to the torsional design of a high-rise building example. All three sampling approaches are performed and their results are compared. Very accurate metamodels are generated and work well for the example problem.

2 OPTIMIZATION FORMULATION

To find an optimum design, the structural design problem needs to be formulated mathematically and solved using a numerical technique. The goal of design is to minimize an objective function (e.g., a cost function), as

\[ C(\mathbf{x}) \] 

subject to the following constraint functions

\[ \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \]  

\[ \mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^u \]  

where all the design variables are included in a vector, \( \mathbf{x} \). The constraint functions \( \mathbf{g}(\mathbf{x}) \) are included in the design formulation based on applicable design requirements. For practical structural engineering problems, \( \mathbf{g}(\mathbf{x}) \) is implicit and an explicit expression is usually not available. However, \( \mathbf{g}(\mathbf{x}) \) value can be evaluated using structural analyses software, such as an FE code.

3 OPTIMIZATION TECHNIQUE

A brief presentation of the proposed optimization technique is given in this section. This includes the RBF-HDMR method and various sampling schemes adopted in the study.

3.1 The HDMR Framework

A function can be expanded using the HDMR framework, as (Chowdhury et al. 2009)

\[ \mathbf{g}(\mathbf{x}) = g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{1 \leq i < j \leq m} g_{ij}(x_i, x_j) + \sum_{1 \leq i < j < k \leq m} g_{ijk}(x_i, x_j, x_k) + \cdots + g_{12...m}(x_1, x_2, \ldots, x_m) \]  

(4)

Eq. (4) is essentially a combination of component functions, and \( g_0 \) and \( g_i(x_i) \) in Eq. (4) can be written as

\[ g_0 = g(\mathbf{c}) \]  

(5)
\[ g_i(x_i) = g(x_i, c^i) - g_0 = g(c_1, \ldots, c_{i-1}, x_i, c_{i+1}, \ldots, c_m) - g(c) \]  

where \( x = c \) in Eq. (5) is a reference point to expand the original function, \( g(x) \). The function value \( g(c) \) at the reference point is referred to as the zeroth order component function. \( g_i(x_i) \) in Eq. (6) is a first-order component function of \( x_i \). Therefore, Eq. (4) can be rewritten as

\[ g(x) = \sum_{i=1}^{m} g(x_i, c^i) - (m - 1)g(c) + \mathbb{R}_2 \]  

where \( \mathbb{R}_2 \) is a residual function which includes the contributions from all higher-order HDMR component functions. If \( \mathbb{R}_2 \) is neglected in Eq. (7), a first-order HDMR model is obtained.

### 3.2 Augmented Radial Basis Functions (RBFs)

Augmented RBFs are improved RBFs in which linear or quadratic functions are added to standard RBFs, in order to improve accuracy for approximating low-order functions (Fang and Horstemeyer 2006, Hassing et al. 2010). An augmented RBF is written as shown in Eq. (8).

\[ \tilde{g}(x) = \sum_{j=1}^{n} \lambda_j \phi(\|x - x_j\|) + \sum_{k=1}^{p} r_k f_k(x) \]  

where the two terms of Eq. (8) are the basic RBF and augmented polynomial functions, respectively.

### 3.3 RBF-HDMR Method

Replace \( g(x_i, c^i) \) in Eq. (7) by its augmented RBF metamodel, a first-order RBF-HDMR model can be expressed as

\[ \tilde{g}(x) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n_i} \lambda_{ij} \phi\left(\|x_i - x_j\|\right) + \sum_{k=1}^{p_i} r_{ik} f_k(x_i) \right) - (m - 1)g(c) \]  

For a more accurate RBF-HDMR model, contributions from higher-order component functions shall be included. To achieve this, \( \mathbb{R}_2 \) in Eq. (7) shall be written and added in Eq. (9). The augmented RBF of \( \mathbb{R}_2 \) is expressed as shown in Eq. (10).

\[ \mathbb{R}_2(x) = \sum_{j=1}^{n_{\mathbb{R}_2}} \lambda_{xj} \phi(\|x - x_j\|) + \sum_{k=1}^{p_{\mathbb{R}_2}} r_{xk} f_k(x) \]  

### 3.4 Sampling Approaches

The proposed optimization approach and its efficiency depends on the sampling scheme adopted in the creation of the metamodels, since the required number of FE analyses is essentially the sample size in optimization. In this study, three different sampling approaches are adopted and tested:

(i) A static sampling approach, as shown in Figure 1.

(ii) An iterative sampling approach, as shown in Figure 2. In each design iteration, additional samples are generated surrounding the optimum design point of the previous design iteration. The additional sample size is selected to be twice of the size of the design variable vector.

(iii) An iterative sampling approach, as shown in Figure 2. Only one additional sample is generated at the optimum design point of the previous design iteration.

Both iterative sampling approaches start with initial sample points for first-order HDMR models. An NLP solver is used to find an optimal point based on the metamodels. Additional
sample points are determined, and corresponding FE analyses are performed at these points, before numerical optimization algorithm is applied again. This process is continued, until the objective function value converges. An improved RBF-HDMR model is achieved due to the addition of the residual function $R_2$ based on the additional sample points located and added in the design iterations.

Figure 1. Static sampling approach.

Figure 2. Two iterative sampling approach.

4 A CONCRETE BUILDING EXAMPLE

An eighteen-story concrete building structure is optimized in this section. The structural plan of a typical floor is shown in Figure 3. The building has two towers and they are connected on top of the building. In the numerical optimization problem, the shear wall thicknesses are selected as design variables in structural optimization, i.e., $w_i$ ($i = 1, \ldots, 3$), as illustrated in Figure 3. To reduce the large amount of torsion in the building, two structural optimization formulations are developed and presented in Table 1, where $T_1$ and $T_i$ are the first translational period and first torsional period of the structure, respectively.
Figure 3. Shear wall layout and design variables.

Table 1. Two optimization formulations.

<table>
<thead>
<tr>
<th></th>
<th>Formulation 1</th>
<th>Formulation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>$C(w_{i=1,3}) = \frac{T_i}{T_1}$</td>
<td>$C(w_{i=1,3}) = \sum_{i=1}^{3} L_i w_i$</td>
</tr>
<tr>
<td>subject to</td>
<td>$0.4 \leq w_{i=1,3} \leq 0.8 \text{ m}$</td>
<td>$g(w_{i=1,3}) = \frac{T_i}{T_1} - 0.85 \leq 0$</td>
</tr>
<tr>
<td></td>
<td>$0.4 \leq w_{i=1,3} \leq 0.8 \text{ m}$</td>
<td></td>
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</table>

Table 2. Optimization results of the building example.

<table>
<thead>
<tr>
<th>HDMR method</th>
<th>Static sampling approach</th>
<th>Iterative sampling approach</th>
<th>Iterative sampling approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulation</td>
<td>Formulation 1</td>
<td>Formulation 2</td>
<td>Formulation 1</td>
</tr>
<tr>
<td>$w_1$ (m)</td>
<td>0.800</td>
<td>0.425</td>
<td>0.800</td>
</tr>
<tr>
<td>$w_2$ (m)</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
</tr>
<tr>
<td>$w_3$ (m)</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
</tr>
<tr>
<td>$T_i / T_1$ (HDMR)</td>
<td>0.669</td>
<td>0.750</td>
<td>0.672</td>
</tr>
<tr>
<td>$T_i / T_1$ (FE)</td>
<td>0.669</td>
<td>0.750</td>
<td>0.669</td>
</tr>
<tr>
<td>$T_i / T_1$ (% error)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.5%</td>
</tr>
<tr>
<td>No of iterations</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>No of samples</td>
<td>$13+12 = 25$</td>
<td>$13+12 = 25$</td>
<td>$13+6 = 19$</td>
</tr>
</tbody>
</table>

SAP2000 (Computer and Structures Inc. 2011), an existing FE analysis software package, is used to build and analyze the structure model. Thirteen initial sample points (i.e., FE models) are generated using five different thickness values of each shear wall. A first-order HDMR model is created using these initial sample points. In the static sampling approach, additional 12 samples are added. For the iterative sampling approaches, six and one additional sample points are generated in each subsequent design iteration. Table 2 lists the design optimization results based on all three sampling approaches. Using the final shear wall thicknesses given in Table 2, the optimal designs are further verified using SAP2000 analysis results. The predicted errors of $T_i / T_1$ using HDMR models are very small compared with the FE results for all three sampling approaches. For the two iterative sampling approaches, only one or two additional iterations are
necessary to obtain convergence of the optimization process. The iterative sampling approach using only one additional sample point is shown to be most efficient, requiring the least numbers of FE analyses to find the optimal designs. In this example, all three sampling approaches work very well and demonstrate to be both efficient and effective.

5 CONCLUDING REMARKS

In this paper, a new optimization technique is developed and applied to building structure optimization. The optimization technique is to integrate RBF-HDMR metamodels and a numerical optimization solver. Three different sampling approaches have been developed and compared, including a static sampling approach and two iterative sampling approaches. All three approaches work well and are able to find similar designs, and small numbers of FE analyses are required. The iterative approach is shown to be very efficient in the optimization of the example building problem.

References