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STABILITY ANALYSIS OF VEHICLE BEHAVIOR IN A LONG PLATOON

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A vehicle platoon is the method of the group of vehicles traveling in short inter-vehicle distance. It can increase the traffic capacity in keeping the traffic safety. Since, in the platoon of vehicles, the vehicles travel in short inter-vehicle distance, the traffic safety depends on the vehicle distance. In this paper, the vehicle velocity is controlled according to the car-following models. The vehicles traveling at the rear of the platoon refer to their frontal vehicles for controlling their velocity. The stability analysis of the models leads the adequate range of the model parameters in which vehicles can control the vehicle distance safely without crush to the preceding vehicle. The stability analysis reveals that the stable region of the parameters becomes wider according to the increase of the number of the frontal vehicles. Finally, the simulation results show that the stability analysis can give the adequate value of the model parameter.

Keywords: Vehicle platoon, Car-following model, Simulation, Mathematical model.

1 INTRODUCTION

A vehicle platoon is the method of the group of the vehicles traveling in short inter-vehicle distance. The traffic capacity of the road can be increased safely if the vehicle distance can be controlled adequately. The smaller the vehicle distance is, the greater the traffic capacity is. When the vehicles in the platoon travel safely, the smallest vehicle distance depends on their velocity control model. In the study, the velocity control model is defined as the car-following model such as Chandler model, optimal velocity (OV) model and so on. In this study, the extended model of OV model is used to control of velocity of the vehicles in the vehicle platoon. When a long vehicle platoon is composed of some short platoons, the stability analysis of the vehicle velocity is performed to determine the condition for the model parameters in the safe traveling of the vehicles. After the analysis, the adequate values of the parameters are determined, and the validity is discussed in the simulation.

2 MATHEMATICAL MODEL

The velocity control model of the vehicles in a long platoon is given in this section. Then, the stability analysis is performed for determining the range of the model parameters.

2.1 Car-Following Model

A car-following model is an equation of motion that controls the velocity or acceleration of a vehicle from the information of the preceding vehicle from the vehicle.

One of the simple models is Chandler model (Chandler 1956), which is given as follows.

$$\ddot{x}_n(t+\Delta t) = a\{\dot{x}_{n+1}(t) - \dot{x}_n(t)\} \quad (1)$$

where x_n is the position of the vehicle n, a is reaction sensitivity of the vehicle performance, and Δt is the delay time. Eq. (1) is an equation of motion that controls an appropriate acceleration in proportion to the relative velocity difference between the vehicle and the preceding vehicle.

Chandler model cannot represent the generation of jammed layers and the abrupt phase transition between low-density and high-density phases.

One of the models that can solve this problem and can express the phase transition of congestion occurrence is the Optimal Velocity Model (OV model) proposed by Bando *et al.* (1995) as shown in Eq. (2) and Eq. (3).

$$\ddot{x}_n(t) = a\{V(\Delta x_n) - \dot{x}_n(t)\} \quad (2)$$

$$\Delta x_n = x_{n+1}(t) - x_n(t) \quad (3)$$

The function $V(\Delta x_n)$ is given as in Eq. (4).

$$V(\Delta x_n) = \frac{V_{max}}{2}[\tanh(\Delta x_n - x_c) + \tan x_c] \quad (4)$$

where x_c is the parameter.

The OV model refers to the vehicle distance $\Delta x_n = x_{n+1}(t) - x_n(t)$ between the preceding vehicle and the following vehicle, and the acceleration $\ddot{x}_n(t)$ is calculated from the difference between the optimal velocity (OV) function $V(\Delta x_n)$ and the vehicle velocity $\dot{x}_n(t)$. The OV function is modeled to approach the ideal speed according to the safe vehicle distance between the preceding vehicle and the following vehicle (Figure 1).

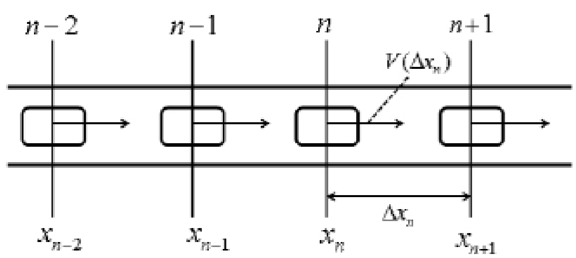


Figure 1. OV model.

2.2 Car-Following Model for Vehicle Platoon

Car-following models usually control the velocity according to the behavior of only one preceding vehicle. Drivers can observe normally not only one preceding vehicle but also the other vehicles. If the information from some vehicles is available, the velocity can be controlled more appropriately.

A multi-leader vehicle type OV model is presented by Lenz *et al.* (1995) in Eq. (5) and Eq. (6).

$$\ddot{x}_n(t) = \sum_{j=1}^{m} a_j\{V(\Delta x_j) - \dot{x}_n(t)\} \quad (5)$$

$$\Delta x_j = \frac{x_{n+j}(t) - x_n(t)}{j} \quad (6)$$

A multi leader-vehicle type platooning model based on OV model is proposed in this study because the stability of the model increases by referring to not only one preceding vehicle but also two or more preceding vehicles (Figure 2).

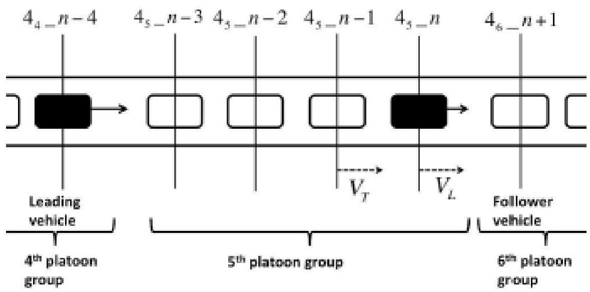


Figure 2. Vehicle platoon.

Kurata *et al.* present the way using the optimal velocity functions with different parameters to the leading and following vehicles in a platoon (Kurata *et al.* 2001). Their model is given as follows in Eq. (7).

$$\begin{aligned} \ddot{x}_{Nm}(t) &= a_L\{V_L(\Delta \mathrm{x}_{Nm}(t)) - \dot{x}_{Nm}(t)\} \quad &\text{(Leading)} \\ \ddot{x}_{Nm-(N-1)}(t) &= a_{Fj}\{V_T(\Delta \mathrm{x}_{Nm-'N-1)}(t)) - \dot{x}_{Nm-(N-1)}(t)\} \quad &\text{(Following)} \end{aligned} \tag{7}$$

where a_L and a_{Fj} are sensitivities for a leading and a following vehicles. The optimal velocity functions for a leading and the following vehicles are given as follows in Eq. (8).

$$\begin{aligned} V_L(\Delta \mathrm{x}) &= \frac{V_{max}}{2}[\tanh(\Delta x - 2x_c) + \tan(2x_c)] \quad &\text{(Leading)} \\ V_{Fj}(\Delta \mathrm{x}) &\frac{V_{max}}{2}[\tanh(\Delta x - x_c) + \tan x_c] \quad &\text{(Following)} \end{aligned} \tag{8}$$

In the above model, each vehicle controls its velocity according to the information from the preceding vehicle.

A velocity control model based on the OV model using more than one leader vehicles is proposed in this study, because the stability of the model increases by referring to not only one vehicle but also two or more vehicles in front. The models for leading and following vehicles are given as follows in Eq. (9).

$$\begin{aligned} \ddot{x}_{Nm_n}(t) &= a_L\{V_L(\mathrm{x}_{N(m+1)_{n+1}}(t) - \mathrm{x}_{Nm_n}(t)) - \dot{x}_{Nm_n}(t)\} \quad &\text{(Leading)} \\ \ddot{x}_{Nm_{n-N}}(t) &= \sum_{j=1}^{m} a_{Fj}\{V_{Tj}(\mathrm{x}_{Nm_{n-N+j}}(t) - \mathrm{x}_{Nm_{n-N}}(t)) - \dot{x}_{Nm_{n-N}}(t)\} \quad &\text{(Following)} \end{aligned} \tag{9}$$

where a_L and a_{Fj} are sensitivities for a leading and a following vehicles. The optimal velocity functions are given as follows in Eq. (10).

$$\begin{aligned} V_L(\Delta \mathrm{x}) &= \frac{V_{Lmax}}{2}[\tanh(\Delta x - 2x_c) + \tan x_c] \quad &\text{(Leading)} \\ V_{Fj}(\Delta \mathrm{x}) &\frac{V_{Fmax}}{2}[\tanh(\Delta x - jx_c) + \tan x_c] \quad &\text{(Following)} \end{aligned} \tag{10}$$

where V_{Lmax} and V_{Fmax} are maximum velocities for a leading and a following vehicles, which are satisfied as follows in Eq. (11).

$$V_{Lmax} < V_{Fmax} \tag{11}$$

2.3 Stability Analysis

The sensitivity relationship is assumed as follows in Eq. (12).

$$a_j \leq a_1 \tag{12}$$

Consider a disturbance y_n near the steady state $x = x_0$ as shown in Eq. (13).

$$x_n = x_0 + y_n = bn + ct \quad |y_n| \ll 1 \qquad b = \frac{L}{N}, c = V(b) \tag{13}$$

Substituting equation (11) to E (7) leads to Eq. (14).

$$\ddot{y}_{n-m}(t) = \sum_{j=1}^{m} a_j \{ f(\frac{y_{n+j-m}(t) - y_{n-m}(t)}{j}) - \dot{y}_{n-m}(t) \} \tag{14}$$

Suppose the following solution is given as the Fourier series of y_n shown in Eq. (15).

$$y_k(n,t) = \exp(i\alpha_k n + zt) \qquad \alpha_k = \frac{2\pi}{N} k \tag{15}$$

Substituting equation (13) into equation (12) leads to Eq. (16).

$$z^2 + f \sum_{j=1}^{m} a_j \frac{1-\cos(\alpha_k j)}{j} - if \sum_{j=1}^{m} a_j \frac{\sin(\alpha_k j)}{j} + z \sum_{j=1}^{m} a_j = 0 \tag{16}$$

When the real part of z is negative ($u < 0$), the value of y_k is stable because it converges to 0 over time. Therefore, the real part of z must be negative ($u < 0$) to keep the steady flow stable. When the real part of z is 0, Eq. (10) is in a neutral stable state (Eq. 17).

$$(-p^2 + f\sigma_c) + i(p\sigma_\alpha - f\sigma_s) = 0 \tag{17}$$

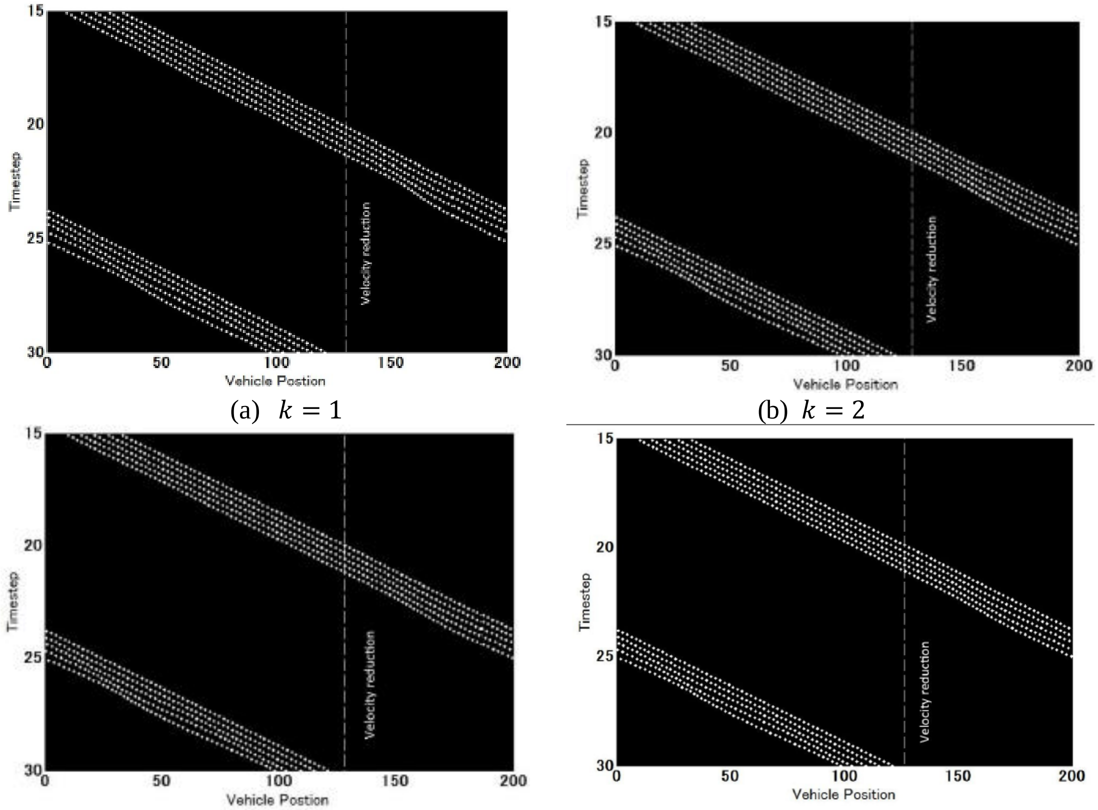
The following system of equations is obtained in Eq. (18) and Eq. (19).

$$-p^2 + f\sigma_c = 0\,, p\sigma_\alpha - f\sigma_s = 0 \tag{18}$$

$$\frac{f}{\sigma_\alpha^2} = \frac{\sigma_c}{\sigma_s^2} \tag{19}$$

3 SIMULATION

A platoon of five vehicles runs on a one-lane road. The platoon has one lead vehicle and four following vehicles. Periodic boundary conditions are applied at the left and right edges of the road. In the periodic boundary condition, the vehicle exiting from the right end appears on the left end, so it is like going around a circuit. It is performed for 60 seconds with a time step of 0.1 second, up to a total of 600 time-steps. In single-vehicle platooning, all vehicles refer to the preceding vehicle to determine acceleration. On the other hand, in the four-vehicle reference platooning, the leading vehicle and the first following vehicle always refer to one vehicle. For the second following vehicle, refer to 2 vehicles, for the third following vehicle, refer to 3 vehicles, and for the fourth following vehicle, refer to 4 vehicles.



(a) $k = 1$ (b) $k = 2$

(c) $k = 3$ (d) $k = 4$

Figure 3. Transition diagram of vehicles in platoon.

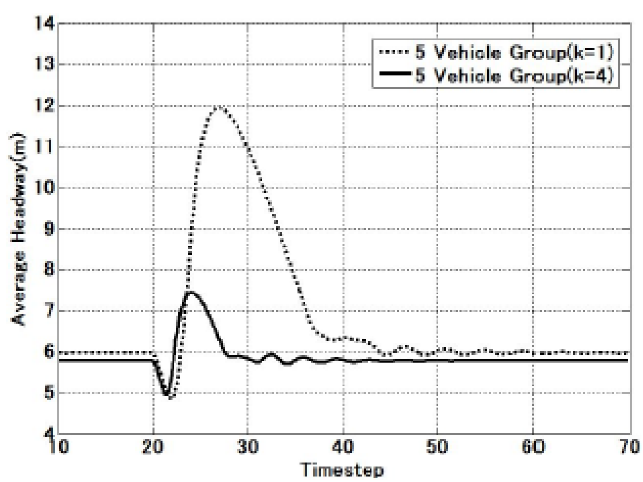


Figure 4. Comparison of average vehicle headways.

The transition diagrams of platooning with one leader vehicle, two leader vehicle, three leader vehicle and four leader following models are shown in Figure 3 (a), (b), (c), and (d), respectively. The vertical and the horizontal axes denote the timestep and the coordinate of vehicles, respectively.

The average vehicle distances of the following vehicles are shown in Figure 4. The figure is plotted with the average vehicle distance of following vehicles excluding the leading vehicle as the vertical axis and the time step as the horizontal axis, respectively. The dashed line is the average vehicle distance of one-leader vehicle following model and the solid line is the average vehicle distance of four-lead vehicle following model. In single-vehicle platooning, all vehicles refer to the preceding vehicle to determine acceleration. On the other hand, in the four-vehicle reference platooning, the leading vehicle and the first following vehicle always refer to one vehicle. For the second following vehicle, refer to 2 lead vehicles, for the third following vehicle, refer to 3 vehicles, and for the fourth following vehicle, refer to 4 vehicles. When entering the deceleration section, the average inter-vehicle distance for both the one-vehicle reference platooning and the four-vehicle platooning decreases to 4.8 m at around 220 time-steps. It rises to 12m. On the other hand, the average vehicle distance of 4-vehicle platooning remained at 7.4m. In platooning, it is possible to reduce air resistance by sufficiently shortening the distance between trucks. If the vehicle distance fluctuates greatly, there is a risk of collision. In multi-vehicle platooning, the deceleration behavior of the preceding vehicle is sensed earlier, and the speed is reduced, so the fluctuation of the inter-vehicle distance can be suppressed. In the four-vehicle platooning, the safe inter-vehicle distance converges at around 510 steps, while in the one-vehicle platooning, the safe inter-vehicle distance converges at around 700 steps.

4 CONCLUSIONS

A vehicle platoon is the method of the group of the vehicles traveling in short inter-vehicle distance. Safe travel of the vehicles in the vehicle platoon depends on the adequate control of the inter-vehicle distance. In this study, the velocity control model is defined as the car-following model.

When a long vehicle platoon is composed of some short platoons, the stability analysis of the model is performed to determine the condition for the model parameters in the safe traveling of the vehicles. The validity of the proposed model was discussed in the computer simulations. The multi-vehicle platooning model could reduce the fluctuations in acceleration and velocity and in addition, converges the velocity at a steady value faster.

Acknowledgments

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